Global numerical simulations of vortex-mediated pulsar glitches in full general relativity

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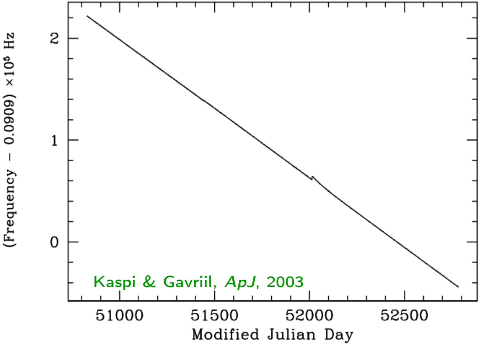
in collaboration with

N. Chamel (ULB), J. Novak (LUTH) & M. Oertel (LUTH)

Sourie, Oertel & Novak, *PRD*, 2016
Sourie, Chamel, Novak & Oertel, submitted to *MNRAS*

1. Introduction
    - Observations
    - Vortex-mediated glitch theory
2. Simulations of pulsar glitches in GR
    - Realistic equilibrium configurations
    - Dynamics of giant glitches
    - Astrophysical considerations
3. Conclusion

The glitch phenomenon



Kaspi & Gavriil, *ApJ*, 2003

Observational features

Espinoza et al., *MNRAS*, 2011

- **amplitude:**

$$\Delta\Omega/\Omega \sim 10^{-11} - 10^{-5}$$

- short **rise time:**

$$\boxed{\tau_r < 30 \text{ s}} \quad \leftarrow\!-- \text{ Vela}$$

- exponential **relaxation** on several days or months.

→ glitch = manifestation of an **internal process**
(except possibly for highly magnetised neutron stars)

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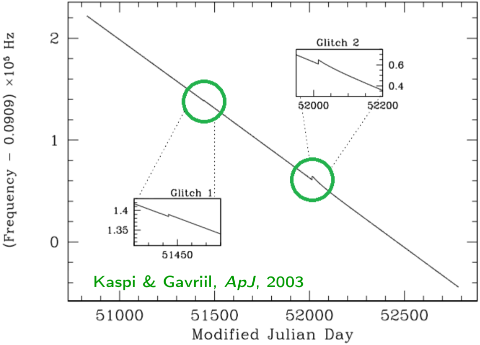

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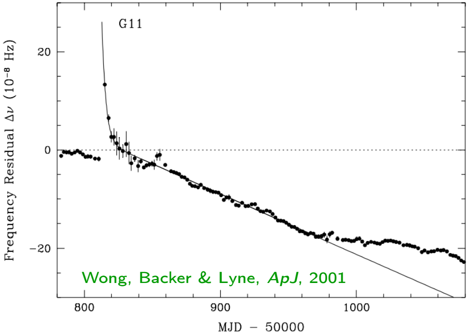

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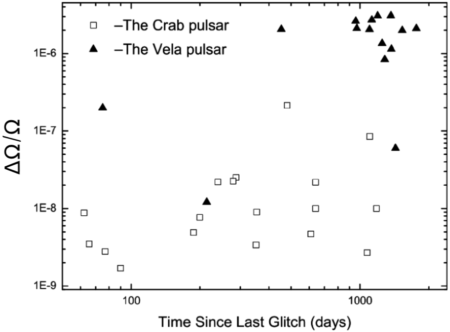
Distinct glitching behaviors

Wang et al., *Ap&SS*, 2012



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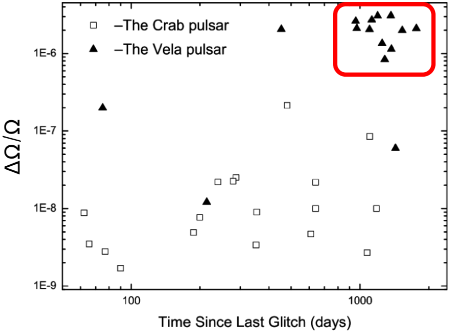

quasi-periodic giant glitches with a very narrow spread in size

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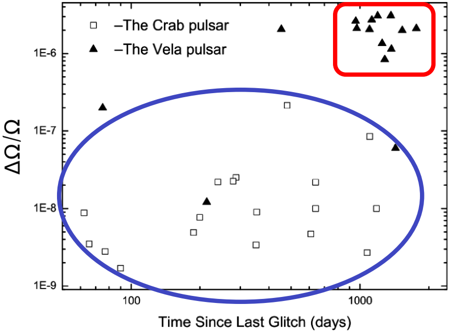

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glitches of various sizes at random intervals of time

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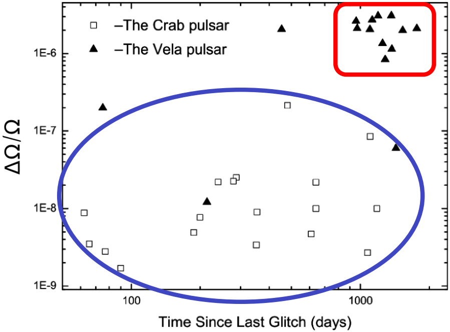

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glitches of various sizes at random intervals of time

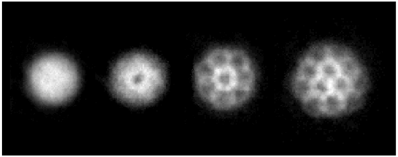
Different models of glitches Haskell & Melatos, *IJMPD*, 2015

- Rearrangement of the moment of inertia --→ crustquakes,
- Angular momentum transfer between *two* fluids --→ **superfluidity**.

Superfluidity in neutron stars

Superfluid properties:

- *zero* viscosity,
- angular momentum quantized into *vortex lines*.



Madison et al., *PRL*, 2000

Theoretical predictions for NSs

$$T \lesssim T_c \sim 10^9 - 10^{10} \text{ K}$$

- **superfluid** neutrons in the core & in the inner crust of NSs.
- **superconducting** protons in the core.

Observational evidence

- *Long relaxation time scales in pulsar glitches,*
- Fast cooling in Cassiopeia A,
- QPOs from SGRs, ...

Vortex-mediated glitch theory

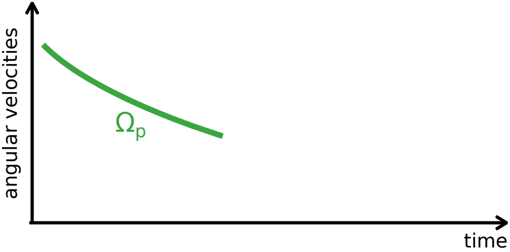
Anderson & Itoh, *Nature*, 1975

Two-fluid model

Baym et al., *Nature*, 1969

- Charged particles:

  $\Omega_{\text{p}} = \Omega \leftrightarrow$ pulsar



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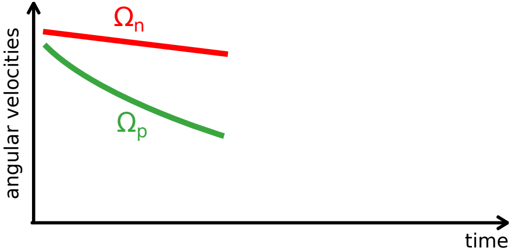
  $\Omega_n \gtrsim \Omega_p$



Key assumption: the vortices can **pin** to the crust and/or to flux tubes.

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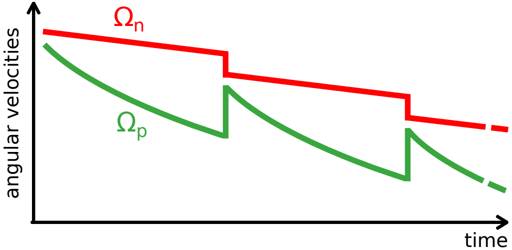

Once a critical lag $\Omega_n - \Omega_p$ is reached:

some vortices get **unpinned** and are allowed to move **radially**

$\dashrightarrow$ angular momentum transfer between the fluids

This work

Question:

What is the impact of **general relativity** on the global dynamics of superfluid neutron stars during a glitch spin-up ?

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$\rightarrow$ global simulations based on a smooth-averaged *hydrodynamical* approach (for Vela: $\sim 10^{17}$ vortices).

This work

Question:

What is the impact of **general relativity** on the global dynamics of superfluid neutron stars during a glitch spin-up ?

→ global simulations based on a smooth-averaged *hydrodynamical* approach (for Vela: $\sim 10^{17}$ vortices).

→ *fundamental hypothesis*:

hydrodynamical time ~ 0.1 ms $\ll$ **rise time** (dissipation)

the glitch event can be well described by a sequence of **quasi-stationary equilibrium** configurations

Assumptions & Ingredients

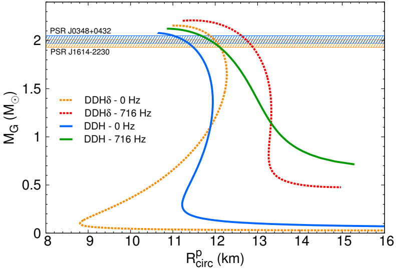
Prix et al., *PRD*, 2005 & Sourie et al., *PRD*, 2016

Equilibrium configurations:

- $T = 0$,
- no magnetic field,
- dissipative effects are **neglected**,
- **uniform** composition: p, e^-, n,
  $\rightsquigarrow$ the crust is not considered,
- asymptotically flat, **stationary**, **axisymmetric** & **circular** metric,
- **rigid-body** rotation: Ω_{n}, Ω_{p}.

Equations of state:

- Polytropic EoSs,
- *Density-dependent RMF models* (DDH & DDHδ).



Fluid couplings

Moments of inertia:

$$\mathrm{d}J_X = I_{XX}\ \mathrm{d}\Omega_X + I_{XY}\ \mathrm{d}\Omega_Y \qquad X, Y \in \{\mathrm{n}, \mathrm{p}\}$$

$$\hat{I}_X = I_{XX} + I_{XY} \qquad\qquad \hat{I} = \hat{I}_\mathrm{n} + \hat{I}_\mathrm{p}$$

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In the slow-rotation approximation ($\Omega_\mathrm{n}, \Omega_\mathrm{p} \ll \Omega_\mathrm{K}$), the fluids are mainly coupled through two *non-dissipative* mechanisms:

- entrainment effect

  due to the strong interactions between nucleons *in the core*:

  $$\boxed{p_X^\alpha = \mathcal{K}^{XX} n_X u_X^\alpha + \mathcal{K}^{XY} n_Y u_Y^\alpha}$$

  Andreev & Bashkin, *SJETP*, 1976

- relativistic frame-dragging effect

  associated with the rotation of the two fluids, Ω_n and Ω_p:

  $$\boxed{g_{t\varphi} \neq 0}$$

  Carter, *Annals of Physics*, 1975

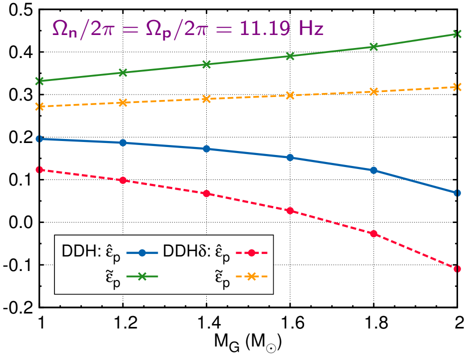
Entrainment VS frame-dragging

Coupling coefficients:

$$\hat{\varepsilon}_X = I_{XY} \,/\, \hat{I}_X$$

In the slow-rotation approximation:

$$\boxed{\hat{\varepsilon}_{\mathrm{p}} = \frac{\tilde{\varepsilon}_{\mathrm{p}} - \varepsilon^{LT}_{\mathrm{n}\to\mathrm{p}}}{1 - \varepsilon^{LT}_{\mathrm{p}\to\mathrm{p}} - \varepsilon^{LT}_{\mathrm{n}\to\mathrm{p}}}}$$



Remarks:

- $\tilde{\varepsilon}_X$ characterizes entrainment,
- in Newtonian gravity:

$$\hat{\varepsilon}_X = \tilde{\varepsilon}_X$$

NB: $\hat{\varepsilon}_{\mathrm{n}} = \hat{I}_{\mathrm{p}}/\hat{I}_{\mathrm{n}} \times \hat{\varepsilon}_{\mathrm{p}} \simeq 0.05 \times \hat{\varepsilon}_{\mathrm{p}}$

Angular momentum transfer

Langlois et al., *MNRAS*, 1998 & Sidery et al., *MNRAS*, 2010

$\Omega_\mathrm{n} - \Omega_\mathrm{p} = \delta\Omega_0 \Rightarrow$ the dynamics is governed by **mutual friction forces**

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Assuming *straight vortices*, the mutual friction moment considered reads

$$\Gamma_{\mathrm{int}} = -\int \frac{\mathcal{R}}{1+\mathcal{R}^2} \Gamma_{\mathrm{n}} n_{\mathrm{n}} \varpi_{\mathrm{n}} \chi_{\perp}^2 \, \mathrm{d}\Sigma \times (\Omega_{\mathrm{n}} - \Omega_{\mathrm{p}}) = -2\bar{\mathcal{B}} \hat{I}_{\mathrm{n}} \Omega_{\mathrm{n}} \zeta \times \delta\Omega$$

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superfluid vorticity

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superfluid vorticity

mean mutual friction parameter

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superfluid vorticity

mean mutual friction parameter

⇝ the geometry of the vortex array and the interactions between superfluid vortices and superconducting flux tubes are **poorly known**.

Spin-up time scale

Evolution equations:

$$\left\{ \begin{array}{lcl} \dot{j}_{\rm n} & = & +\,\Gamma_{\rm int}, \\ \dot{j}_{\rm p} & = & -\,\Gamma_{\rm int}. \end{array} \right. \dashrightarrow \quad \frac{\delta\dot{\Omega}}{\delta\Omega} = -\frac{\hat{I}\hat{I}_{\rm n}}{I_{\rm nn}I_{\rm pp} - I_{\rm np}^2} \times 2\bar{\mathcal{B}}\zeta\Omega_{\rm n}$$

- Theoretical rise time:

$$\rightsquigarrow \quad \delta\Omega(t) = \delta\Omega_0 \times e^{-\frac{t}{\tau_{\rm r}}}$$

$$\boxed{\tau_{\rm r} = \frac{\hat{I}_{\rm p}}{\hat{I}} \times \frac{1 - \hat{\varepsilon}_{\rm p} - \hat{\varepsilon}_{\rm n}}{2\zeta\bar{\mathcal{B}}\Omega_{\rm n}}}$$

- Numerical modelling:

Computation of $\Omega_{\rm n}(t)$ & $\Omega_{\rm p}(t)$ profiles from $\Omega_{\rm n,0} > \Omega_{\rm p,0}$

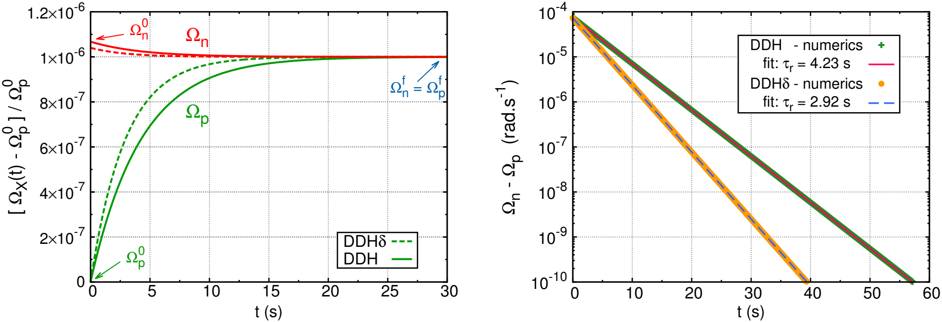
Input parameters

M_G, Ω, $\Delta\Omega/\Omega$, EoS, β-eq., $\bar{\mathcal{B}}$

Time evolution

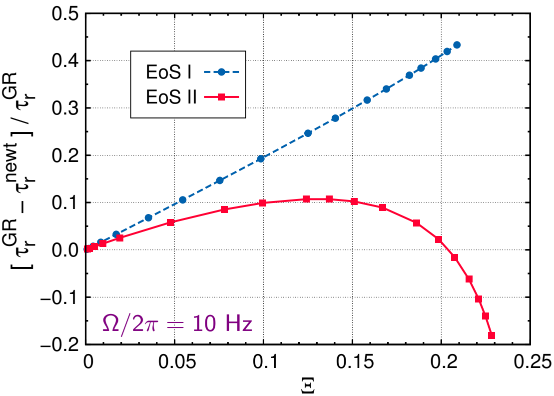
$$\Delta\Omega/\Omega = 10^{-6},\ \Omega_{\rm n}^{f} = \Omega_{\rm p}^{f} = 2\pi \times 11.19\ \text{Hz},$$

$$M_{\rm G} = 1.4\ \text{M}_\odot\ \&\ \bar{\mathcal{B}} = 10^{-4}$$



⇢ the spin-up time scale can be very precisely estimated from *stationary configurations* only.

Influence of general relativity on τ_r



- polytropic EoSs
- **compactness** parameter:

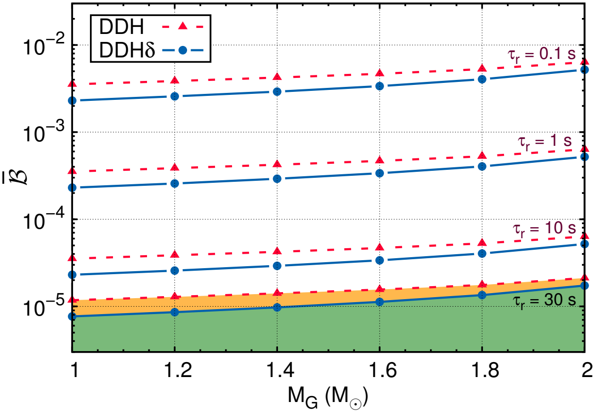
$$\boxed{\Xi = \frac{G\,M_G}{R_{c,eq}\,c^2}}$$

NB: for NSs, $\Xi \simeq 0.2$

- these relative differences also depend on Ω

The Vela pulsar

$$\Delta\Omega/\Omega = 10^{-6},\ \Omega_{\rm n}^f = \Omega_{\rm p}^f = 2\pi \times 11.19\ \text{Hz}$$



- $\bar{\mathcal{B}} \nearrow \implies \tau_{\rm r} \searrow$

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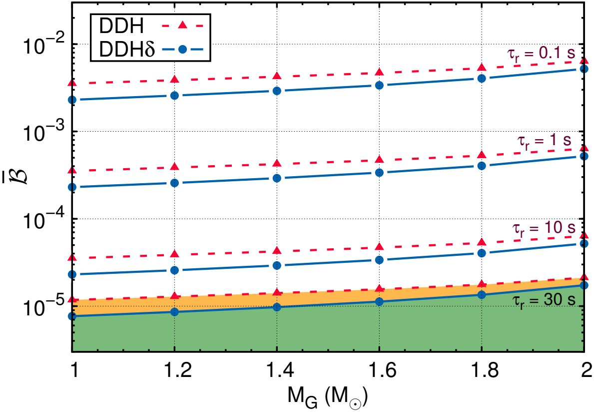

- $\bar{\mathcal{B}} \nearrow \Longrightarrow \tau_r \searrow$
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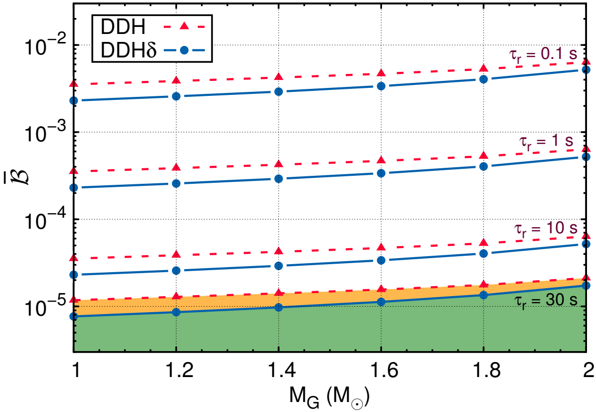

- $\bar{\mathcal{B}} \nearrow \implies \tau_r \searrow$
- *Constraint on* $\bar{\mathcal{B}}$:

  $\tau_r < 30$ s $\Rightarrow \bar{\mathcal{B}} > 10^{-5}$
- $\bar{\mathcal{B}} < 0.5 \rightsquigarrow \tau_r > 0.6$ ms

$\hookrightarrow$ the glitch event is a **quasi-stationary** process

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Conclusion & perspectives

- *Additional coupling* through relativistic frame-dragging effects,
- *Relativistic corrections* on the spin-up time: $\sim 50\%$ (core),

  $\hookrightarrow$ should be included in a quantitative model of glitches.

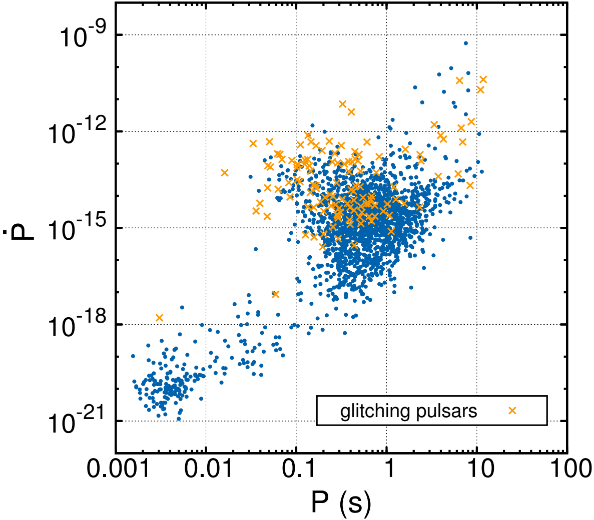
Future work:

- Improve our numerical models by including the crust and considering that only a small amount of vortices is involved in the glitch event,
- Compare with future accurate observations of glitches,
- Include interactions with flux tubes in a more realistic mutual friction moment.

Thank you!

$P - \dot{P}$ diagram

ATNF Pulsar Database ; Manchester et al., *Astron. Journal*, 2005



Glitch activity

Observables

Link, Epstein & Lattimer, *PRL*, 1999

- Average glitch activity:

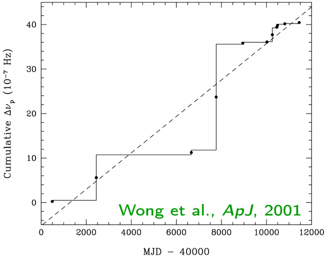
$$\overline{A} = \frac{1}{t_{\text{obs}}} \frac{\sum_i \Delta\Omega_i}{\Omega}$$

- Coupling parameter:

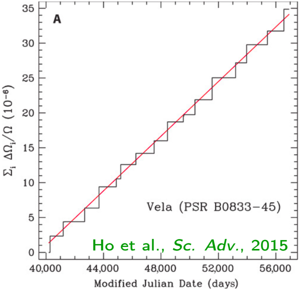
$$\boxed{G = \frac{\Omega}{|\dot{\Omega}|} \times \overline{A}}$$

⇢ **Vela**: $G \simeq 1.62 \times 10^{-2}$

⇢ **Crab**: $G \simeq 1.45 \times 10^{-5}$



Wong et al., *ApJ*, 2001



Ho et al., *Sc. Adv.*, 2015

Spacetime metric

Bonazzola, Gourgoulhon, Salgado & Marck, *A&A*, 1993

Rotating neutron stars, at **equilibrium**, described by $(\mathcal{E}, \boldsymbol{g})$:

- asymptotically flat: $\boldsymbol{g} \to \boldsymbol{\eta}$ at spatial infinity $(r \to +\infty)$,
- stationary & axisymmetric: $\frac{\partial g_{\alpha\beta}}{\partial t} = \frac{\partial g_{\alpha\beta}}{\partial \varphi} = 0$,
- circular: perfect fluids $\Rightarrow$ *purely circular* motion around the rotation axis with Ω_n, Ω_p (+ **rigid rotation**).

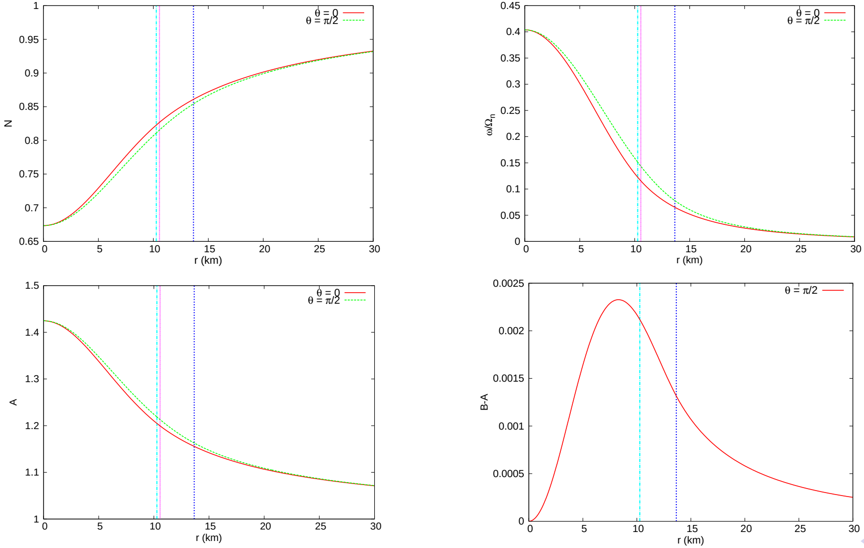
Spacetime metric in quasi-isotropic coordinates:

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -N^2\,\mathrm{d}t^2 + A^2(\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2) + B^2 r^2 \sin^2\theta(\mathrm{d}\varphi - \omega\,\mathrm{d}t)^2$$

At spatial infinity

$$N, A, B \to 1 \;\;\&\;\; \omega \to 0$$

Metric potentials



Relativistic two-fluid hydrodynamics

Carter, "Covariant theory of conductivity in ideal fluid or solid media", 1989 & Carter & Langlois, *Nuc. Phys. B*, 1998

System = two **perfect** fluids:

- superfluid neutrons → $\vec{\boldsymbol{n}}_{\mathrm{n}} = n_{\mathrm{n}} \vec{\boldsymbol{u}}_{\mathrm{n}}$,
- protons & electrons → $\vec{\boldsymbol{n}}_{\mathrm{p}} = n_{\mathrm{p}} \vec{\boldsymbol{u}}_{\mathrm{p}}$.

Energy-momentum tensor

$$T_{\alpha\beta} = n_{\mathrm{n}\alpha} p^{\mathrm{n}}_{\beta} + n_{\mathrm{p}\alpha} p^{\mathrm{p}}_{\beta} + \Psi g_{\alpha\beta}$$

$\hookrightarrow$ **conjugate momenta**

Entrainment matrix:

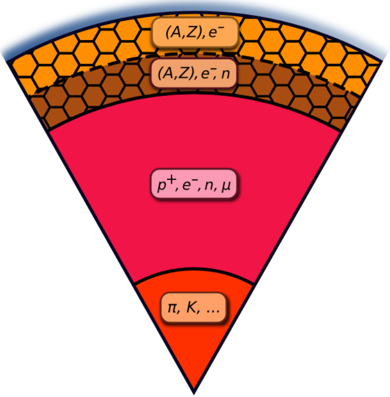
$$\begin{cases} p^{\mathrm{n}}_{\alpha} &= \mathcal{K}^{\mathrm{nn}} n^{\mathrm{n}}_{\alpha} + \mathcal{K}^{\mathrm{np}} n^{\mathrm{p}}_{\alpha} \\ p^{\mathrm{p}}_{\alpha} &= \mathcal{K}^{\mathrm{pn}} n^{\mathrm{n}}_{\alpha} + \mathcal{K}^{\mathrm{pp}} n^{\mathrm{p}}_{\alpha} \end{cases}$$

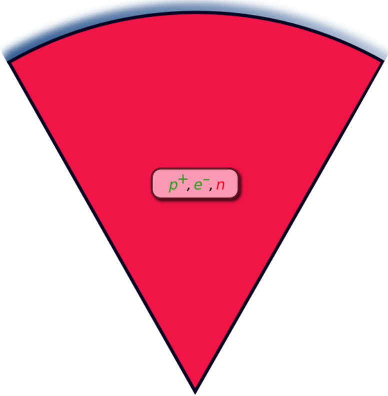
--→ entrainment effect

Equation of state

$$\mathcal{E}(n_{\mathrm{n}}, n_{\mathrm{p}}, \Delta^2)$$

Neutron stars interior

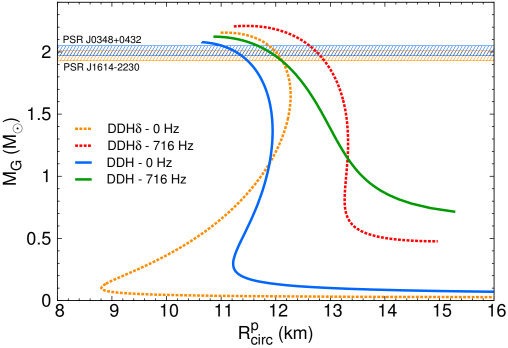




Equations of state

Relativistic Mean-Field Theory:

strong interaction between nucleons ⇔ exchange of effective mesons



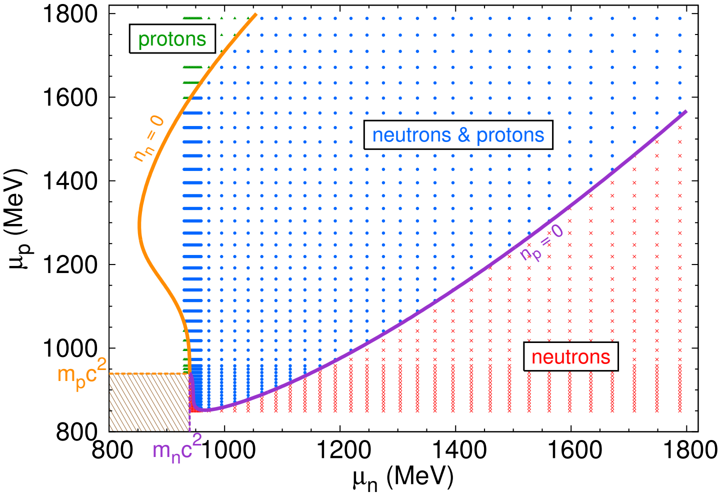
- Gravitational mass:

$$M_G = M^B + E_{bind},$$

- Circumferential radius:

$$R^X_{circ,\ eq} = \mathcal{C}^X / 2\pi .$$

Tabulated EoS



Entrainment effects

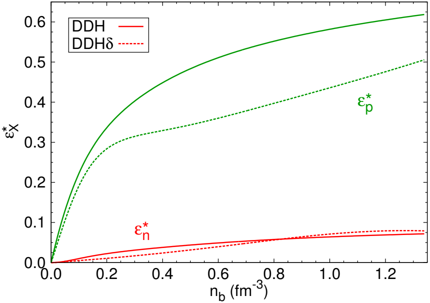
Dynamical effective mass:

$$\boxed{{}^3\vec{\boldsymbol{p}}_{\boldsymbol{X}} = m_X^*\ {}^3\vec{\boldsymbol{u}}_{\boldsymbol{X}}}$$

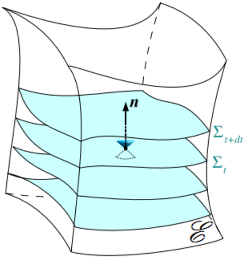
→ in the *rest frame* of the second fluid.

Zero-velocity frame:

$$m_X^* = \overset{\text{special relativity}}{\boxed{\mu^X}} \times \left(1 - \underset{\textbf{entrainment}}{\boxed{\varepsilon_X^*}}\right)$$



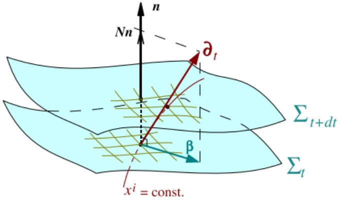
3+1 formalism



Foliation of the spacetime $(\mathcal{E}, \boldsymbol{g})$ by $(\Sigma_t)_{t\in\mathbb{R}}$, with unit normal $\vec{\boldsymbol{n}}$

Eulerian observer $\mathcal{O}_n$: 4-velocity $= \vec{\boldsymbol{n}}$

- lapse function N : $\vec{\boldsymbol{n}} = -N\vec{\boldsymbol{\nabla}} t$,
- shift vector $\vec{\beta}$: $\vec{\boldsymbol{\partial}}_t = N\vec{\boldsymbol{n}} + \vec{\beta}$.



3+1 metric:

$$g_{\alpha\beta}\, \mathrm{d}x^\alpha\, \mathrm{d}x^\beta = -N^2\, \mathrm{d}t^2 + \gamma_{ij} \left(\mathrm{d}x^i + \beta^i\, \mathrm{d}t\right) \left(\mathrm{d}x^j + \beta^j\, \mathrm{d}t\right)$$

Numerical procedure

Paramètres d'entrée :

- une EOS
- H_c^n, H_c^p
- Ω_n, Ω_p

$i = 0$

Initialisation :

- $N = A = B = 1$ et $\omega = 0, \forall\,(r,\theta)$
- $U_n = U_p = 0$
- $H_0^i(r,\theta) = H_c^i\left(1 - \frac{r^2}{R^2}\right)$

Convergence threshold

$$|H^i_{k+1}(r,\theta) - H^i_k(r,\theta)| < \epsilon$$

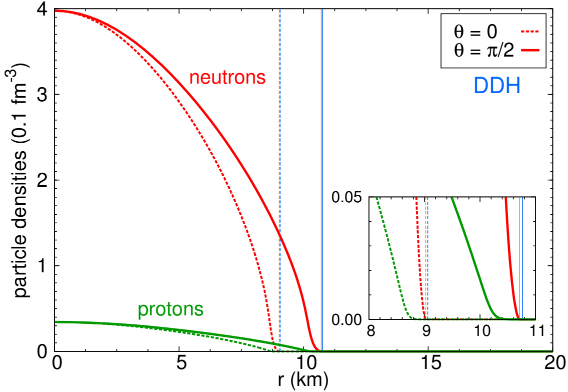
At each iteration

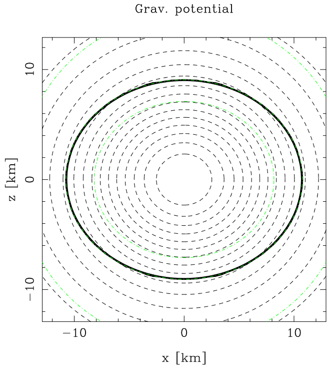
For given values of $(\mu^{\mathrm{n}}, \mu^{\mathrm{p}}, \Delta^2)$, we compute:

1. Ψ, n_{n}, n_{p} and α from the EoS
2. The source terms E, p_φ, $S^i{}_i$,
3. Einstein Equations are solved,
4. Kinetic terms U_{i} et Γ_{i},
5. Computation of H^i_{k+1}.

Density profiles

$$M_G = 1.4\ M_\odot,\ \Omega_n/2\pi = \Omega_p/2\pi = 716\ \text{Hz}$$

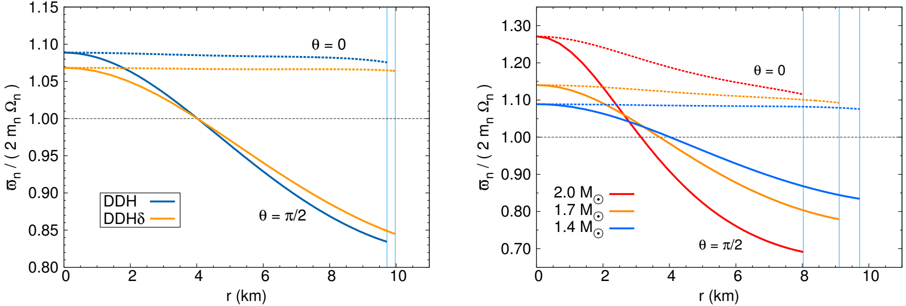




Vorticity

Superfluid vorticity

$$w_{\mu\nu} = \nabla_\mu p^{\mathrm{n}}_\nu - \nabla_\nu p^{\mathrm{n}}_\mu \dashrightarrow \varpi_{\mathrm{n}} = \sqrt{\frac{w_{\mu\nu} w^{\mu\nu}}{2}}$$



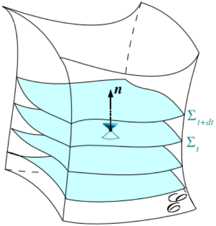
$$\Omega^{\mathrm{n}}/2\pi = \Omega^{\mathrm{p}}/2\pi = 716\ \mathrm{Hz}$$

Angular momenta

Axisymmetry $\leftrightarrow$ $\vec{\chi}$

Komar definition:

$$J_{\mathrm{K}} = -\int_{\Sigma_t} \underbrace{\boldsymbol{T}(\vec{\boldsymbol{n}}, \vec{\chi})}_{-p_\varphi} \, \mathrm{d}^3 V$$



Eulerian observer $\vec{\boldsymbol{n}}$ (3+1)

Angular momentum of each fluid

Langlois, Sedrakian & Carter, *MNRAS*, 1998

$$p_\varphi = \underbrace{\Gamma_{\mathrm{n}} n_{\mathrm{n}} p^{\mathrm{n}}_\varphi}_{j^{\mathbf{n}}_\varphi} + \underbrace{\Gamma_{\mathrm{p}} n_{\mathrm{p}} p^{\mathrm{p}}_\varphi}_{j^{\mathbf{p}}_\varphi}$$

$$J_X = \int_{\Sigma_t} j^X_\varphi \, A^2 B r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\varphi$$

Fluid couplings

In the slow-rotation approximation and to first order in the lag $\delta\Omega = \Omega_\mathrm{n} - \Omega_\mathrm{p}$, the **angular momentum of fluid** X reads

$$\begin{aligned} J_X \quad &\simeq \int_{\Sigma_t} n_X \mu^X \frac{B}{N} \left(\Omega_X - \omega\right) r^2 \sin^2\theta \;\, \mathrm{d}^3V \\ &+ \int_{\Sigma_t} n_X \mu^X \varepsilon_X \frac{B}{N} \left(\Omega_Y - \Omega_X\right) r^2 \sin^2\theta \;\, \mathrm{d}^3V \end{aligned}$$

Introducing $i_X \equiv n_X \mu^X \frac{B}{N} r^2 \sin^2\theta$, we characterize the couplings by

- Entrainment:
- Lense-Thirring:

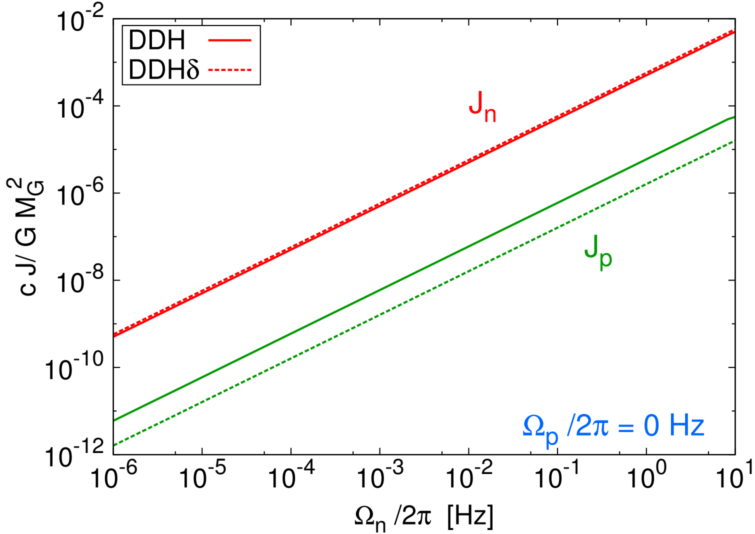
$$\boxed{\tilde{I}_X \;\tilde{\varepsilon}_X \equiv \int_{\Sigma_t} i_X \;\varepsilon_X \;\; \mathrm{d}^3V}$$

$$\boxed{\tilde{I}_X \left(\varepsilon^{LT}_{X\to X}\Omega_X + \varepsilon^{LT}_{Y\to X}\Omega_Y\right) \equiv \int_{\Sigma_t} i_X \;\omega \;\; \mathrm{d}^3V}$$

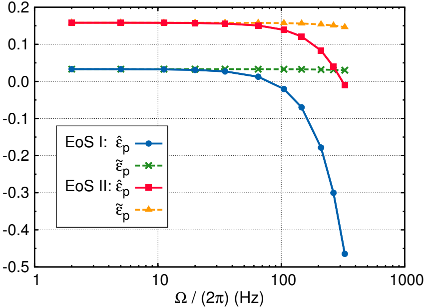
where $\tilde{I}_X \equiv \int_{\Sigma_t} i_X \;\, \mathrm{d}^3V$

$$J_X = \tilde{I}_X \left(1 - \varepsilon^{LT}_{X\to X} - \tilde{\varepsilon}_X\right) \Omega_X + \tilde{I}_X \left(\tilde{\varepsilon}_X - \varepsilon^{LT}_{Y\to X}\right) \Omega_Y$$

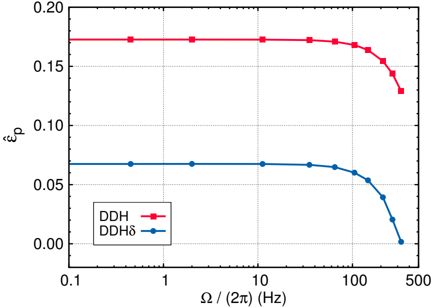
Fluid couplings



Influence of Ω on the couplings



Newtonian gravity



general relativity

Where does the vortex unpinning take place?

Glitches have been generally thought to originate from the crust, because:

- the core superfluid was expected to be strongly coupled to the crust Alpar et al., *ApJ*, 1984
- the analysis of glitch data suggested that the superfluid represents a few percent of the total angular momentum of the star Link et al., *PRL*, 1999

However, this scenario has been recently **challenged**:

- considering entrainment effects, the crust does not carry enough angular momentum Andersson et al., *PRL*, 2012 & Chamel, *PRL*, 2013
- a huge glitch has been observed in PSR 2334+61 Alpar, *AIP Conf.Proc.*, 2011
- the core superfluid could be decoupled from the rest of the star, if vortices are pinned to flux tubes Gügercinoglu & Alpar, *ApJ*, 2014

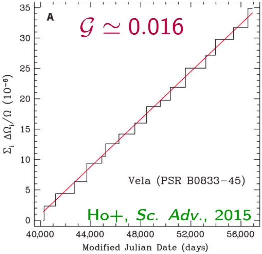
The core superfluid plays a more important role than previously thought.

Additional physical inputs

So far, we assumed that **all** the neutrons can decouple from the protons.

- only a *small fraction* of the neutron fluid could be involved in the glitch:

$$\bar{I}_n / \bar{I} > f \equiv I_n^{nc} / \bar{I} \gtrsim \mathcal{G} \times (1 - \varepsilon_n^{nc})$$



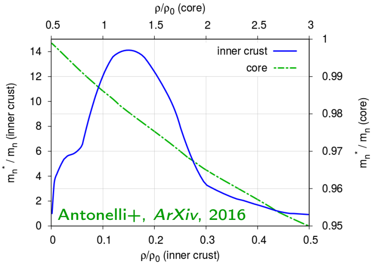
$\mathcal{G} \simeq 0.016$

Ho+, *Sc. Adv.*, 2015

See also: Link+, *PRL*, 1999 & Lyne+, *MNRAS*, 2000

- we need to account for *crustal entrainment* (Bragg scattering):

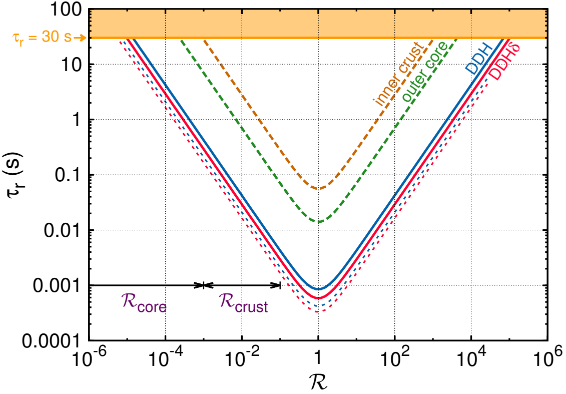
$$-14 \lesssim \varepsilon_n^{nc} \lesssim 0$$



Antonelli+, *ArXiv*, 2016

See also: Chamel, *PRC*, 2012

Constraining the interior of NSs



$$\boxed{\tau_r = \frac{1 - f - \varepsilon_n^{nc}}{2\bar{\mathcal{B}}\Omega_n}}$$

- whole core:

  $f = 0.94, \ \varepsilon_n^{nc} = 0.03$ (DDH)
  $f = 0.96, \ \varepsilon_n^{nc} = 0.02$ (DDHδ)

- outer core:

  $f = 0.016, \ \varepsilon_n^{nc} = 0$

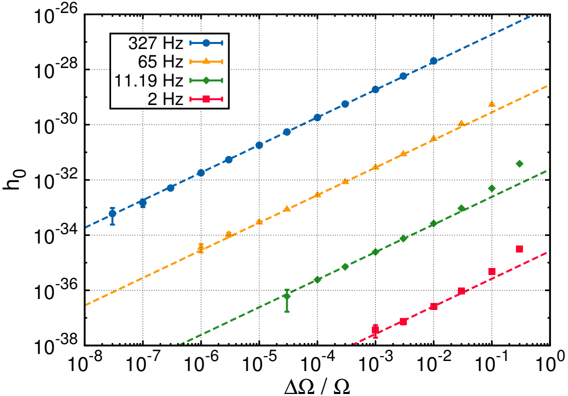
- inner crust:

  $f = 0.064, \ \varepsilon_n^{nc} = -3$

$\tau_r^{min} \simeq 1$ ms - 0.1 s ⇢ the glitch event is a quasi-stationary process ✓

Gravitational wave amplitude

$$h_+(t) = -\frac{3}{2}\sin^2 i\,\frac{G}{Dc^4}\,\ddot{Q} = h_0\sin^2 i\;\, e^{-\frac{t}{\tau_r}}$$



- $D = 1$ kpc,
- $\bar{\mathcal{B}} = 10^{-3}$,
- $M_G = 1.4\ \mathrm{M}_\odot$,
- DDH EoS.

$$h_0 \simeq 1.0\times 10^{-37}\left(\frac{D}{1\ \mathrm{kpc}}\right)^{-1}\left(\frac{\bar{\mathcal{B}}}{10^{-3}}\right)^{2}\left(\frac{\Omega}{10^{2}\ \mathrm{rad.s}^{-1}}\right)^{4}\left(\frac{\Delta\Omega/\Omega}{10^{-6}}\right)$$