Quantum information: from qubits to space-time (part II)

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www.romanorus.com/SouthamptonTutorial2016.pdf

Some reviews

J. Eisert, Modeling and Simulation 3, 520 (2013), arXiv:1308.3318

N. Schuch, QIP, Lecture Notes of the 44th IFF Spring School 2013, arXiv:1306.5551

R. Orus, arXiv:1306.2164, arXiv:1407.6552

J. I. Cirac, F. Verstraete, J. Phys. A: Math. Theor. 42, 504004 (2009)

F. Verstratete, J. I. Cirac, V. Murg, Adv. Phys. 57,143 (2008)

J. Jordan, PhD thesis, www.romanorus.com/JordanThesis.pdf

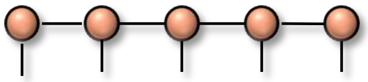
G. Evenbly, PhD thesis, arXiv:1109.5424

U. Schollwöck, RMP 77, 259 (2005)

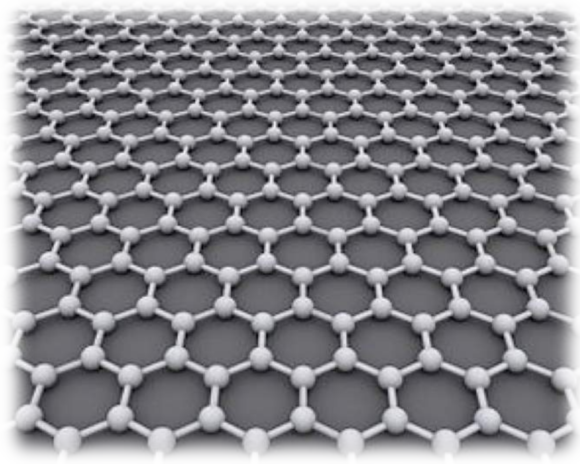
U. Schollwöck, Annals of Physics 326, 96 (2011)

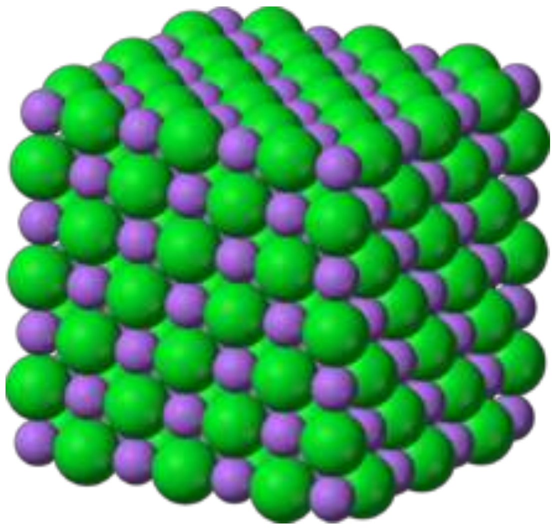
6) Projected Entangled Pair States (PEPS)

From MPS to PEPS

Matrix Product States (MPS)  *1d systems*

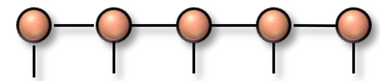
But we want to go beyond 1d systems!!!



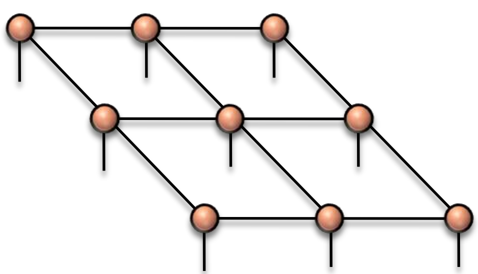


Very painful for DMRG...

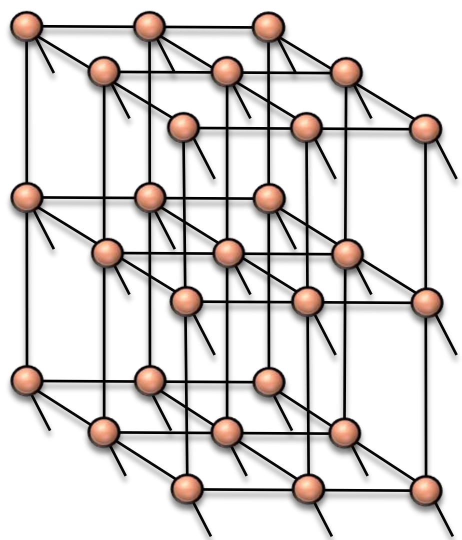
From MPS to PEPS

Matrix Product States (MPS)  *1d systems*

Projected Entangled Pair States (PEPS),
Tensor Product States (TPS)



2d systems

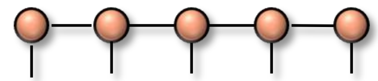


3d systems

and so on…

From MPS to PEPS

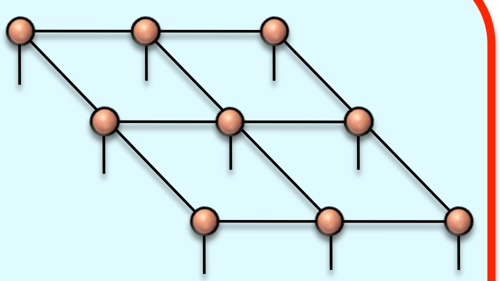
Matrix Product States (MPS)



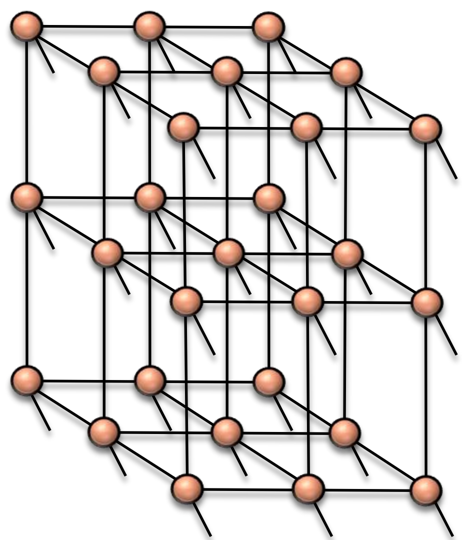
1d systems

Projected Entangled Pair States (PEPS),
Tensor Product States (TPS)

This lecture



2d systems



and so on…

3d systems

Two exact examples

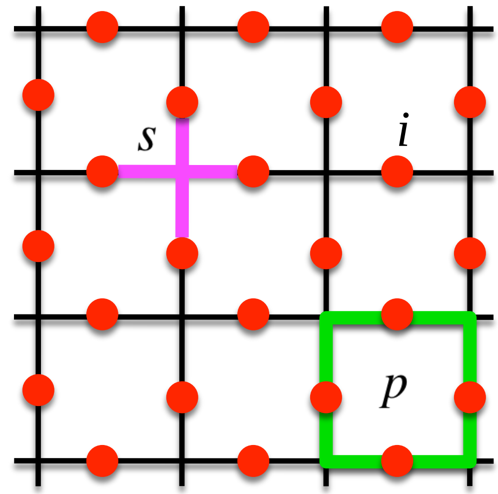
An exact example: Kitaev's Toric Code

Kitaev, 1997

$$H = -J\sum_s A_s - J\sum_p B_p$$

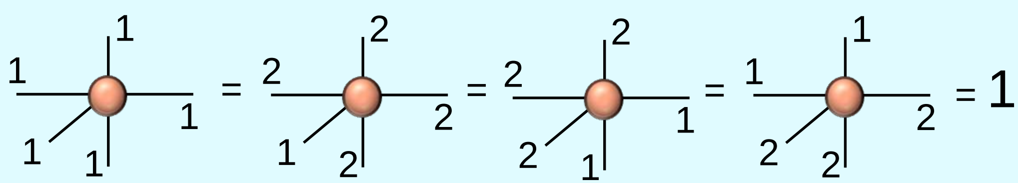
$$A_s = \prod_{i\in s} \sigma_i^x$$ *star operator*

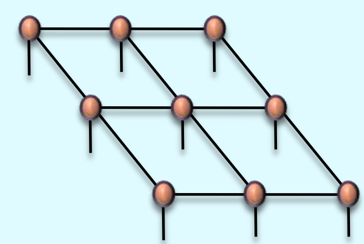
$$B_p = \prod_{i\in p} \sigma_i^z$$ *plaquette operator*



Simplest known model with "topological order"

Ground state (and in fact all eigenstates) are PEPS with D=2

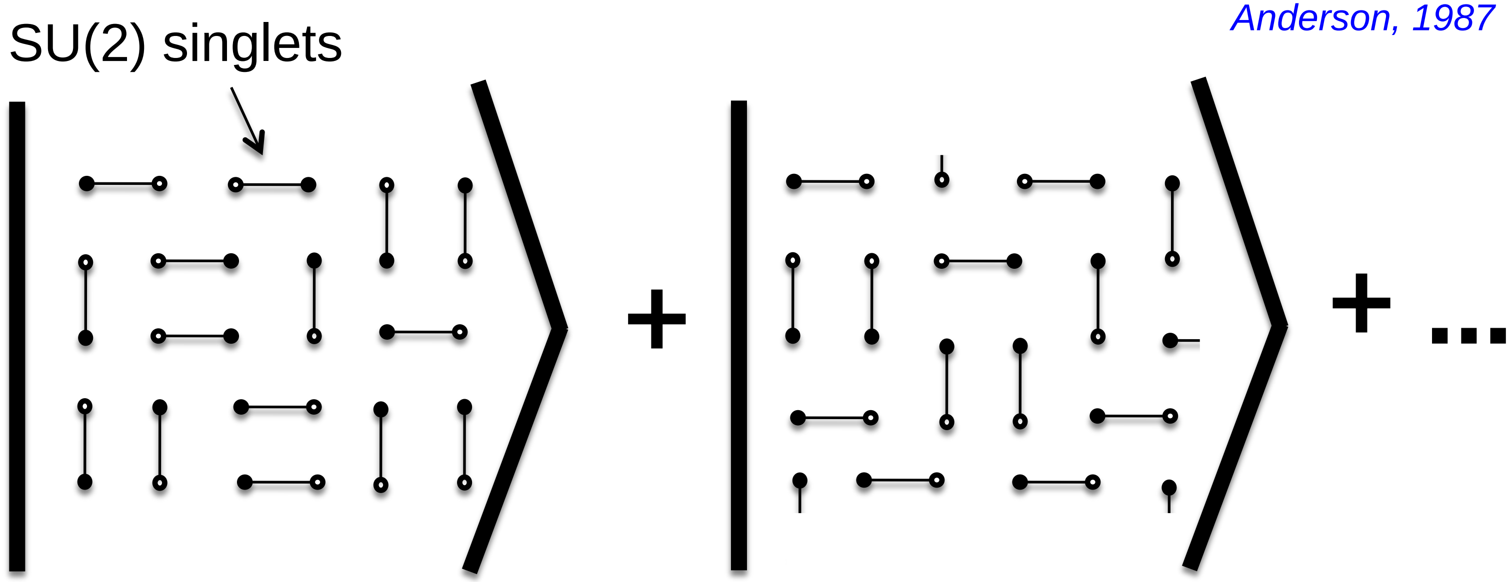




And another tensor rotated 90˚

Resonating Valence Bond State

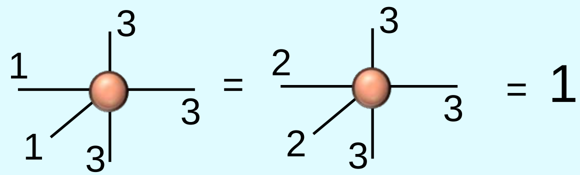
Anderson, 1987



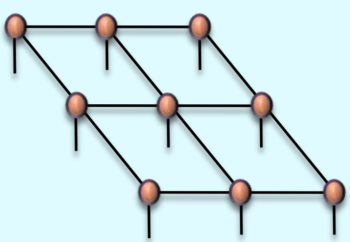
Equal superposition of all possible nearest-neighbor singlet coverings of a lattice (spin liquid)

Proposed to understand high-T_C superconductivity

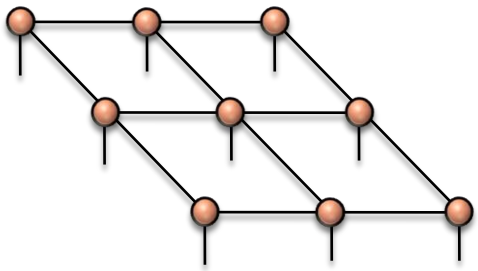
It is a PEPS with D=3



And rotations

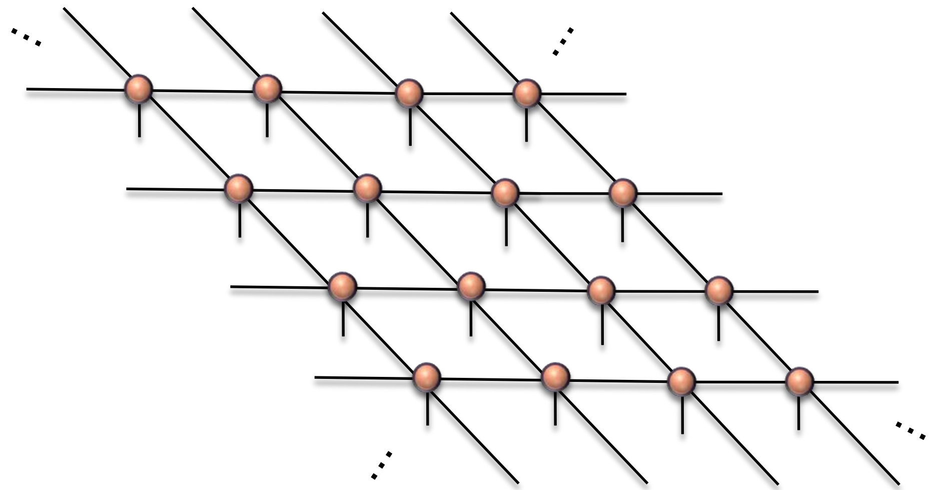


PEPS...



F. Verstraete, I. Cirac, cond-mat/0407066

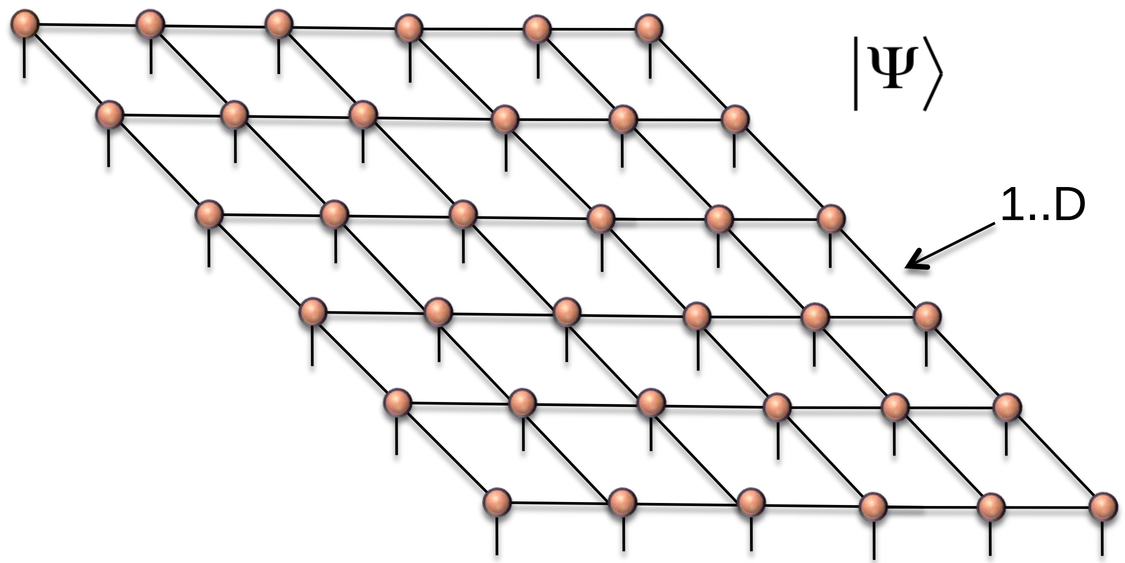
and infinite PEPS (iPEPS)

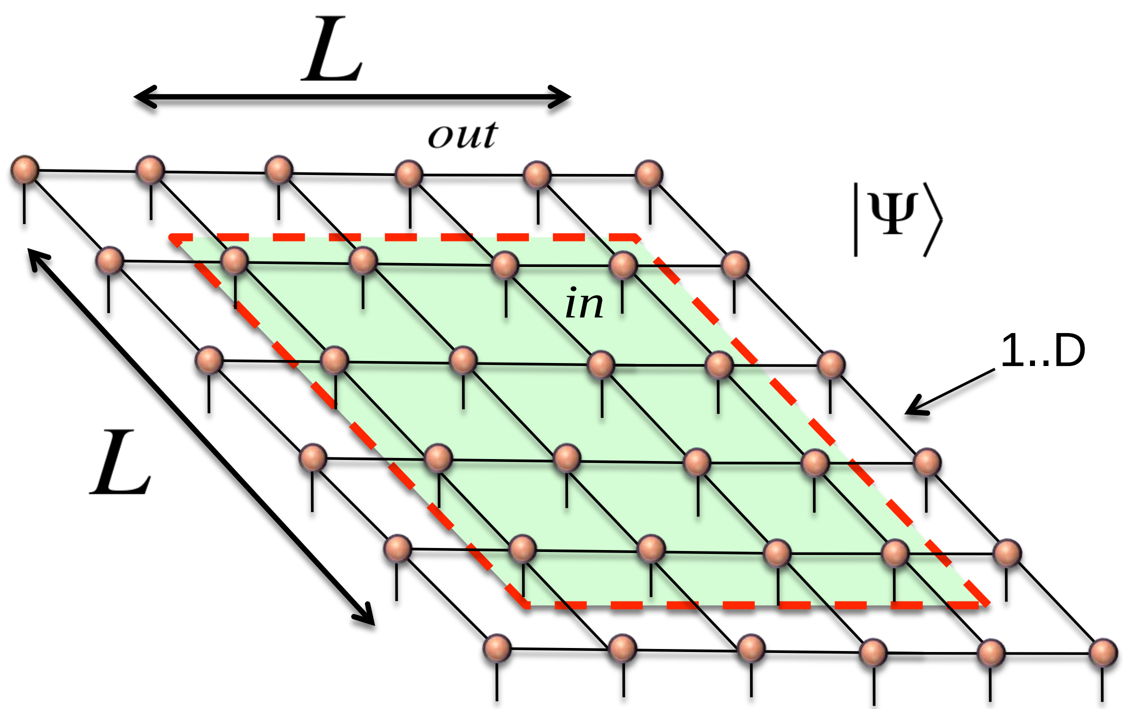


assuming translation invariance

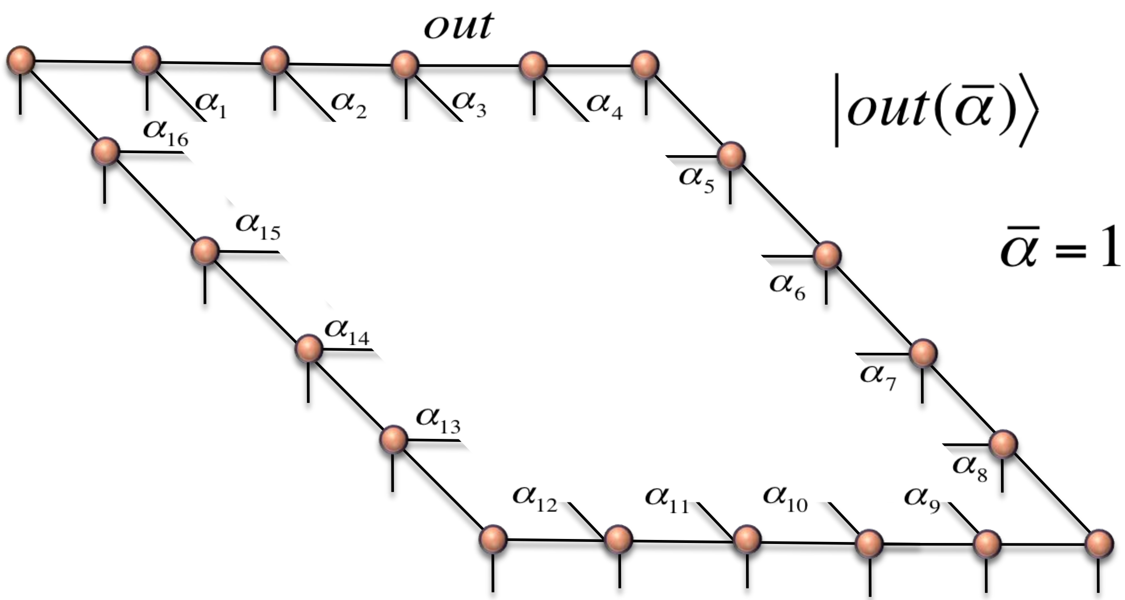
J. Jordan, RO, G. Vidal, F. Verstraete, I. Cirac, Phys. Rev. Lett. 101, 250602 (2008)

PEPS obey 2d area-law


$|\Psi\rangle$
1..D

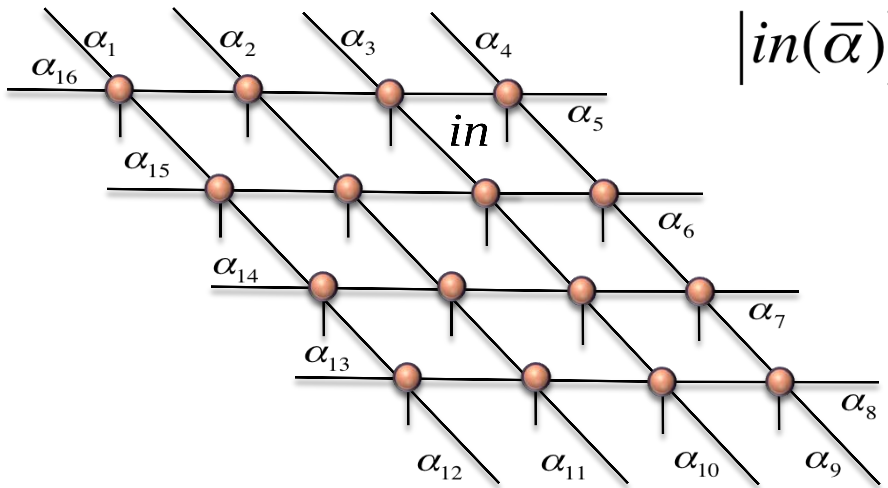


L
out
$|\Psi\rangle$
in
1..D
L



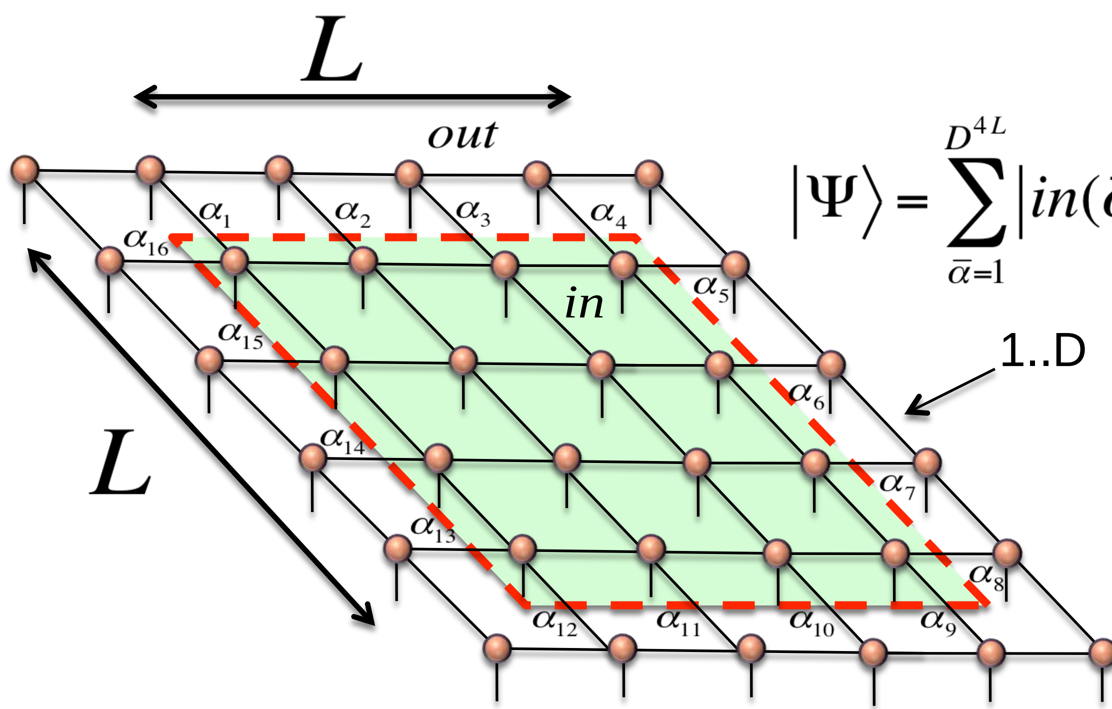
$$|out(\bar{\alpha})\rangle$$

$$\bar{\alpha} = 1, 2, \ldots, D^{4L}$$



$$|in(\bar{\alpha})\rangle$$

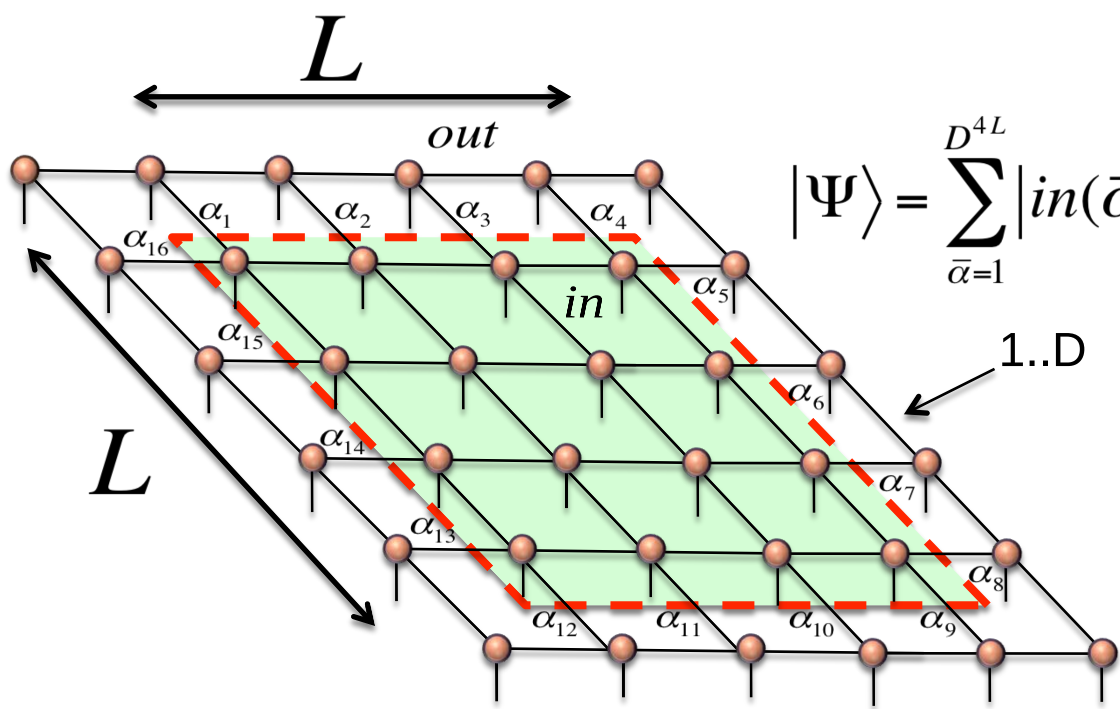
$$\bar{\alpha} = 1, 2, \ldots, D^{4L}$$



$$|\Psi\rangle = \sum_{\bar{\alpha}=1}^{D^{4L}} |in(\bar{\alpha})\rangle |out(\bar{\alpha})\rangle$$

$$\rho_{in} = \mathrm{tr}_{out}\left(|\Psi\rangle\langle\Psi|\right) = \sum_{\bar{\alpha},\bar{\alpha}'} X_{\bar{\alpha},\bar{\alpha}'} |in(\bar{\alpha})\rangle\langle in(\bar{\alpha}')| \qquad X_{\bar{\alpha},\bar{\alpha}'} = \langle out(\bar{\alpha}')|out(\bar{\alpha})\rangle$$

$$rank(\rho_{in}) \le D^{4L} \qquad S(L) = -\mathrm{tr}\left(\rho_{in}\log\rho_{in}\right) \le \log(D)4L$$



$$|\Psi\rangle = \sum_{\bar{\alpha}=1}^{D^{4L}} |in(\bar{\alpha})\rangle |out(\bar{\alpha})\rangle$$

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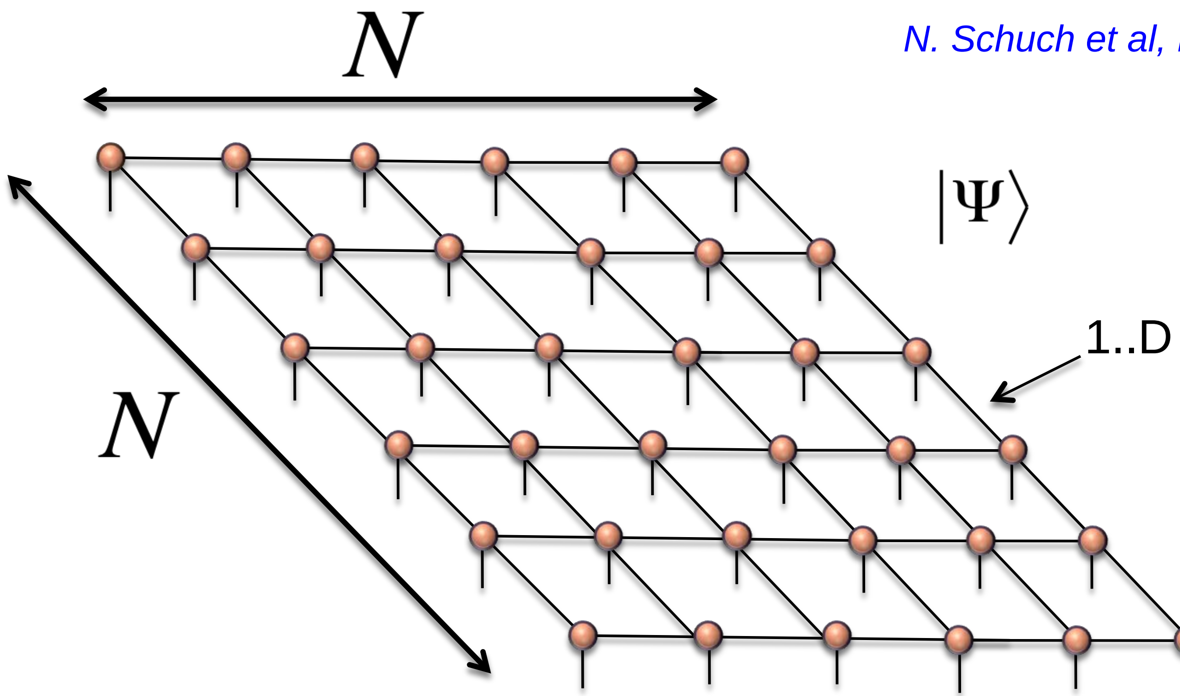
$$rank(\rho_{in}) \le D^{4L} \qquad S(L) = -\mathrm{tr}\left(\rho_{in} \log \rho_{in}\right) \le \log(D)\boxed{4L}$$

prefactor

size of the boundary

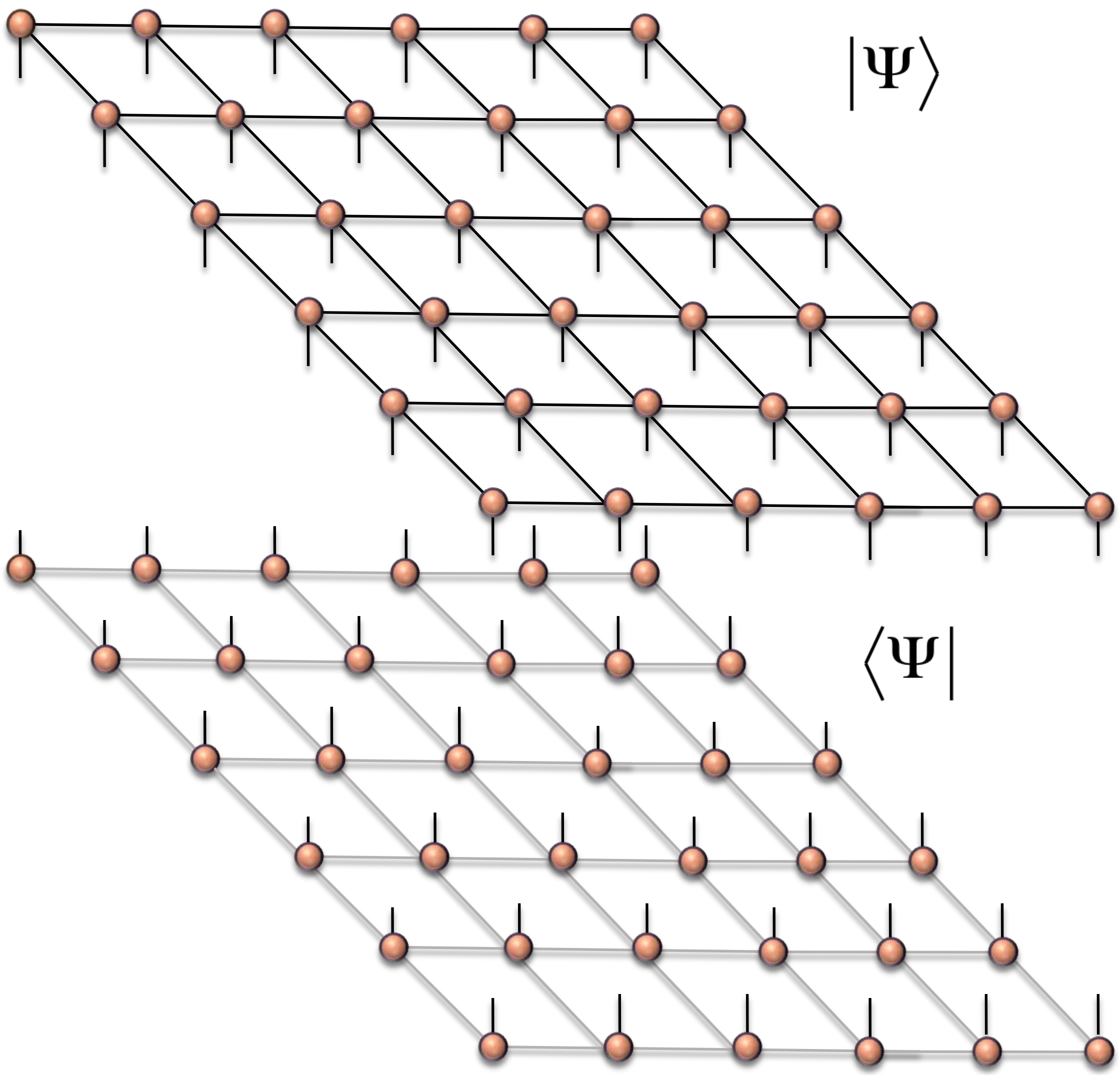
Exact contraction is inneficient

N. Schuch et al, PRL 98, 140506 (2007)



Exact contraction is ineficient

N. Schuch et al, PRL 98, 140506 (2007)



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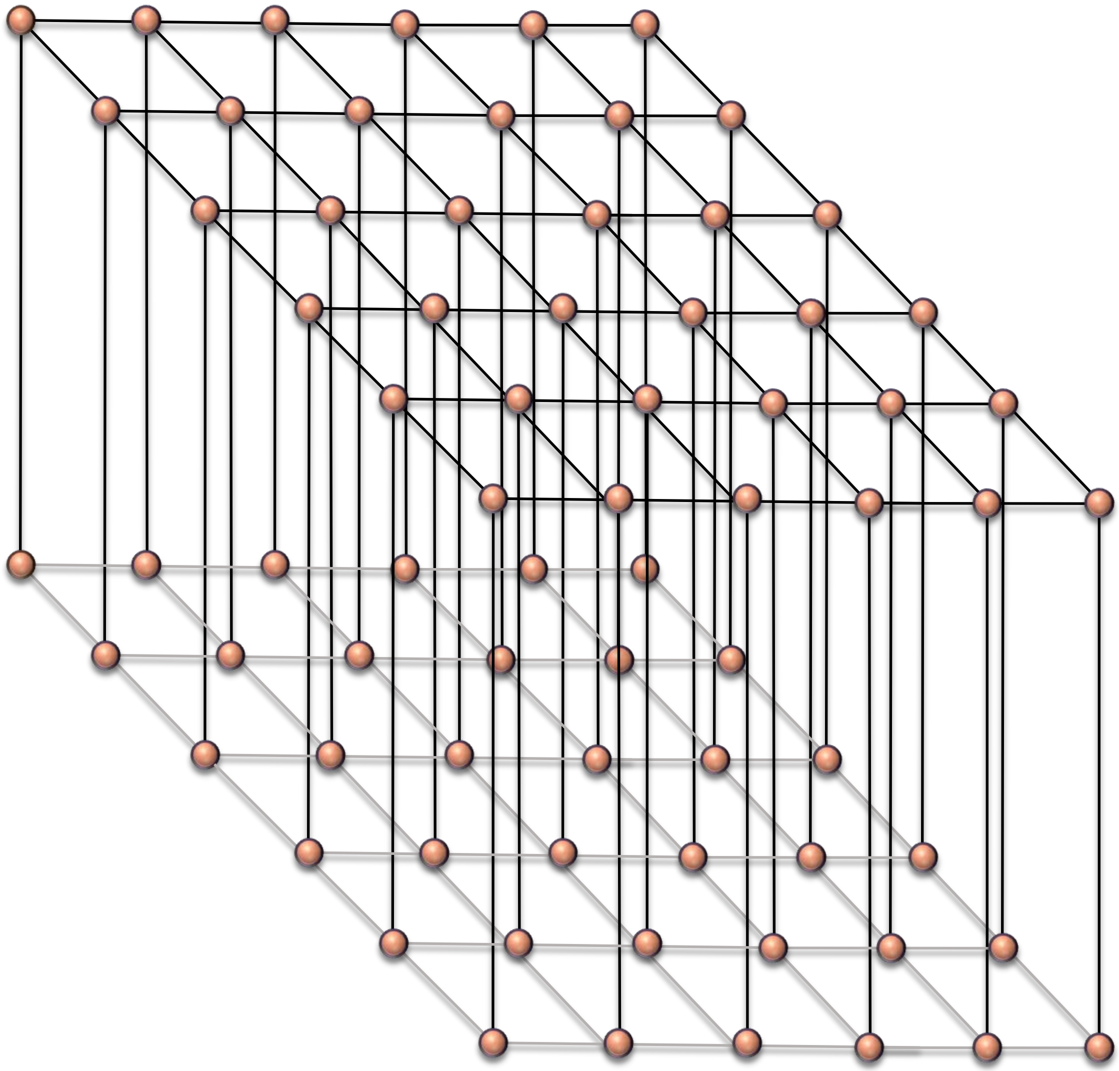
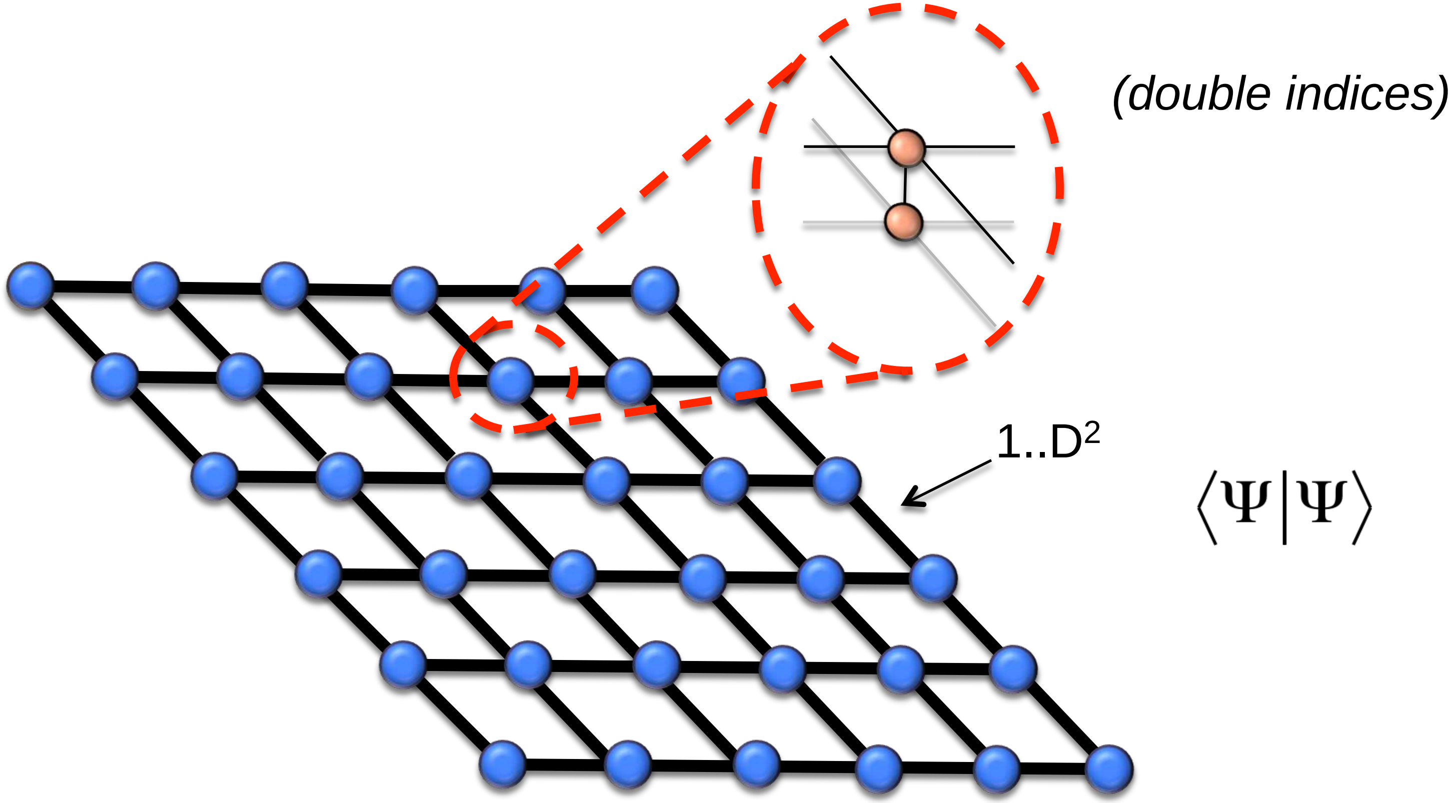


$$\langle \Psi | \Psi \rangle$$

Exact contraction is inneficient

N. Schuch et al, PRL 98, 140506 (2007)



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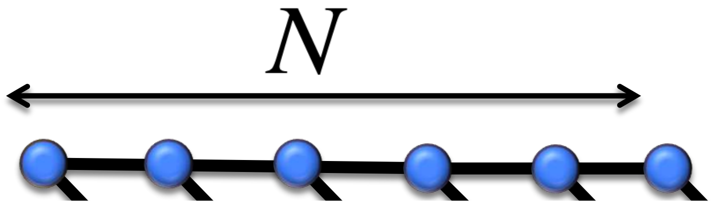


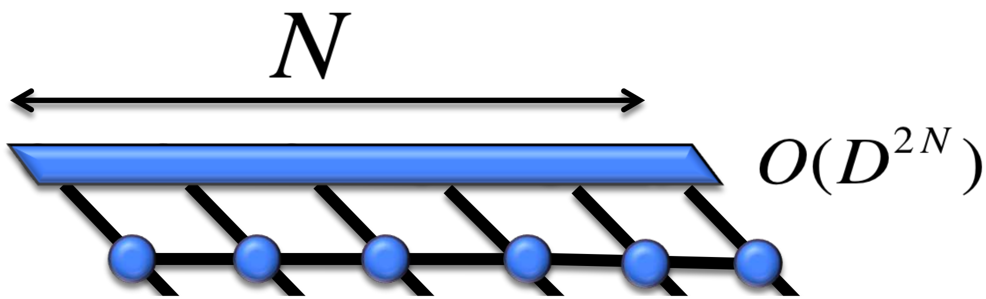
Exact contraction is ineficient

N. Schuch et al, PRL 98, 140506 (2007)

$O(D^{2N})$

Exact contraction is inneficient

N. Schuch et al, PRL 98, 140506 (2007)



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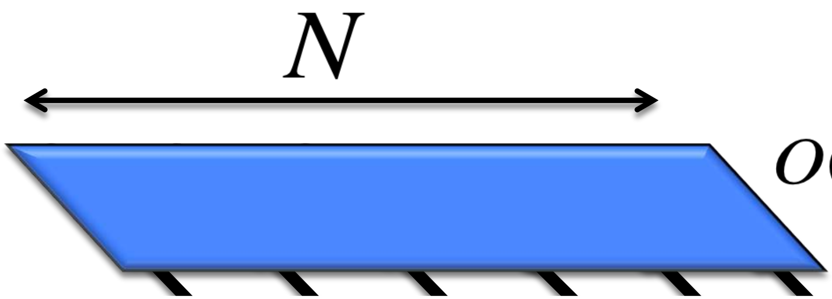
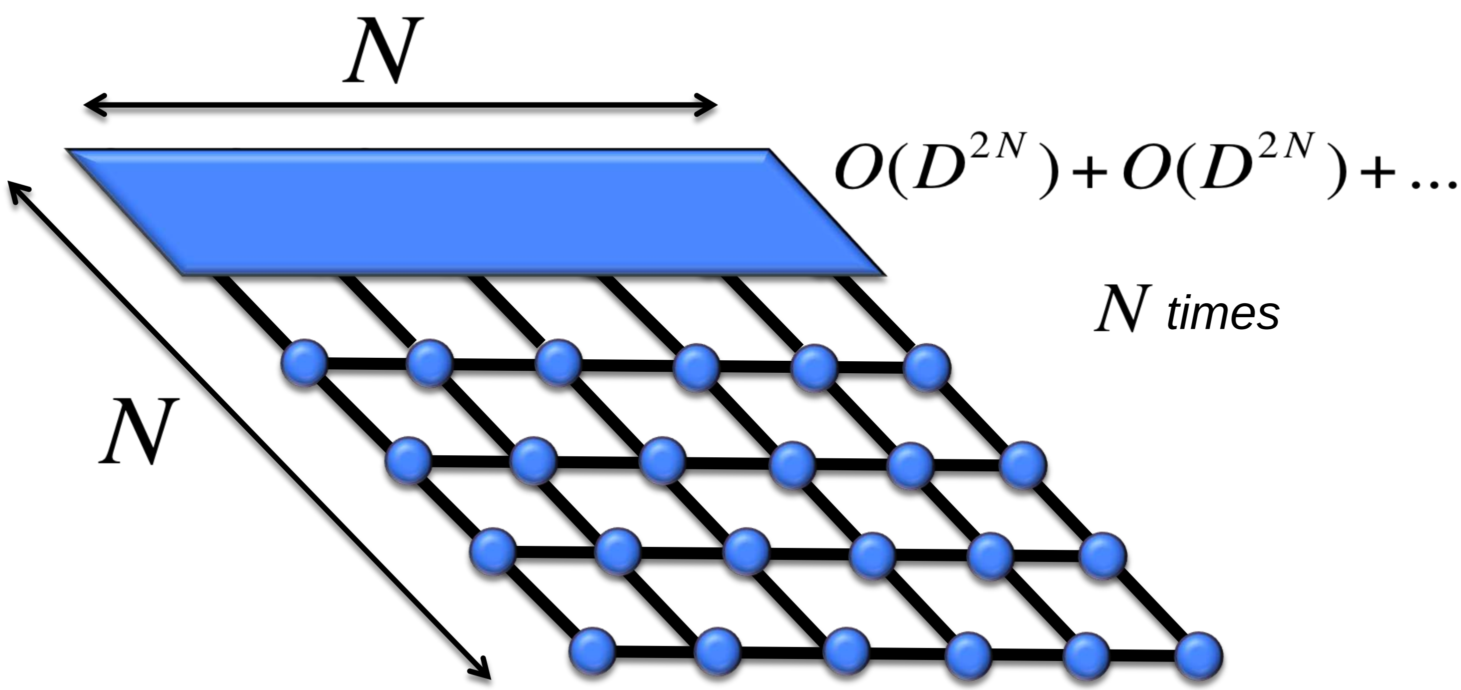
N. Schuch et al, PRL 98, 140506 (2007)



$$O(D^{2N}) + O(D^{2N})$$

Exact contraction is ineficient

N. Schuch et al, PRL 98, 140506 (2007)



computing time ~ $O(ND^{2N})$ **Exponential amount of time!**

Mathematical statement: exact contraction of a PEPS is a #P-Hard problem (harder than NP-Complete)

Applies also to expectation values of observables

Critical correlation functions

F. Verstraete et al, PRL 96, 220601 (2006)

$$\left|\Psi(\beta)\right\rangle = \frac{1}{\sqrt{Z(\beta)}} \exp\left(\frac{\beta}{2}\sum_{\langle i,j\rangle} \sigma_z^i \sigma_z^j\right)\left|+,+\ldots+\right\rangle$$

Expectation values are those of the classical 2d Ising model

$$\left\langle \sigma_z^r \sigma_z^{r'} \right\rangle_\beta = \frac{1}{Z(\beta)} \sum_{\{s\}} s^r s^{r'} \exp\left(\beta \sum_{\langle i,j\rangle} s^i s^j\right) \qquad s = \pm 1$$

Critical correlation functions

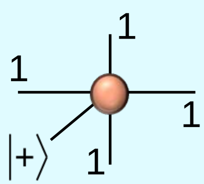
F. Verstraete et al, PRL 96, 220601 (2006)

$$|\Psi(\beta)\rangle = \frac{1}{\sqrt{Z(\beta)}} \exp\left(\frac{\beta}{2} \sum_{\langle i,j \rangle} \sigma_z^i \sigma_z^j \right) |+,+\dots+\rangle$$

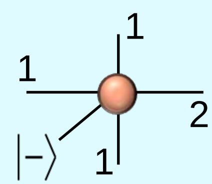
Expectation values are those of the classical 2d Ising model

$$\langle \sigma_z^r \sigma_z^{r'} \rangle_\beta = \frac{1}{Z(\beta)} \sum_{\{s\}} s^r s^{r'} \exp\left(\beta \sum_{\langle i,j \rangle} s^i s^j \right) \qquad s = \pm 1$$

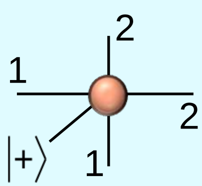
It is a PEPS with D=2 *(left as exercise):*



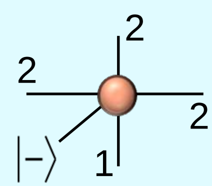
$= \left(\cosh(\beta/2)\right)^4$



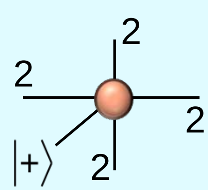
$= \left(\cosh(\beta/2)\right)^3 \left(\sinh(\beta/2)\right)$



$= \left(\cosh(\beta/2)\right)^2 \left(\sinh(\beta/2)\right)^2$



$= \left(\cosh(\beta/2)\right) \left(\sinh(\beta/2)\right)^3$



$= \left(\sinh(\beta/2)\right)^4$

+ permutations

At $\beta_c = \left(\log(1+\sqrt{2})\right)/2$ ***the correlation length is infinite:*** $\langle \sigma_z^r \sigma_z^{r'} \rangle_{\beta_c} \approx \frac{a}{|r-r'|^{1/4}}$

PEPS to/from Hamiltonians

PEPS to/from Hamiltonians

World of Hamiltonians

World of states

PEPS to/from Hamiltonians

World of Hamiltonians

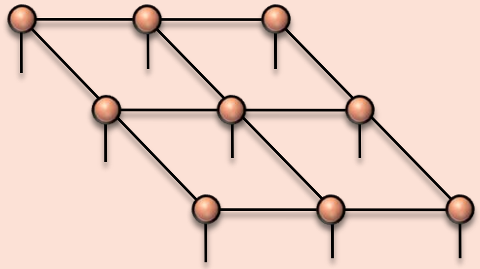
Ground state of a gapped Parent Hamiltonian with local interactions

$$H_{parent} = \sum h_{local}$$

e.g. D. Perez-Garcia et al, QIC 8, 0650 (2008)

World of states

Generic PEPS with finite D



PEPS to/from Hamiltonians

World of Hamiltonians

Ground state of a gapped Parent Hamiltonian with local interactions

$$H_{parent} = \sum h_{local}$$

Hamiltonian with local interactions

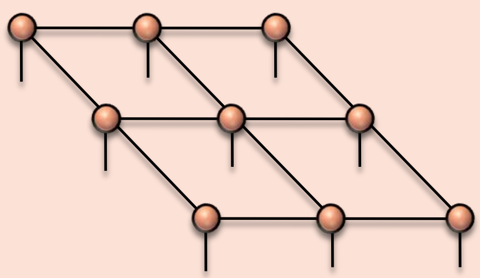
$$H' = \sum h'_{local}$$

e.g. D. Perez-Garcia et al, QIC 8, 0650 (2008)

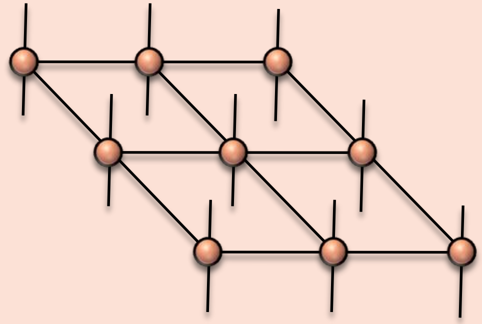
M. Hastings, PRB 73, 085115 (2006)

World of states

Generic PEPS with finite D

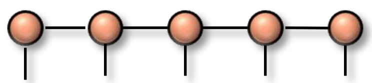


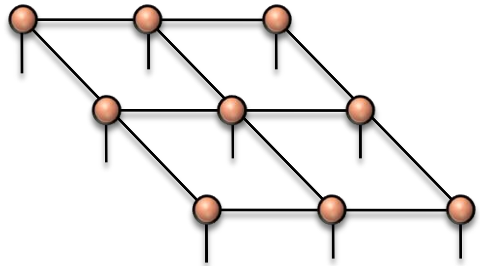
Thermal states can be approx. by a PEPS with finite D



PEPS target the relevant corner of the Hilbert space (area-law)

MPS vs PEPS





MPS in 1d

1d area law $S(L) = O(1)$

Exact contraction is efficient

Finite correlation length

to/from 1d Hamiltonians

PEPS in 2d

2d area law $S(L) = O(L)$

Exact contraction is inefficient

Finite and infinite correlation lengths

to/from 2d Hamiltonians

PEPS as ansatz: variational optimization

Variational optimization (e.g. finite PEPS)

e.g. F. Verstraete, I. Cirac, cond-mat/0407066

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (as in DMRG)

Variational optimization (e.g. finite PEPS)

e.g. F. Verstraete, I. Cirac, cond-mat/0407066

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (as in DMRG)

$|\Psi\rangle$

E

Variational optimization (e.g. finite PEPS)

e.g. F. Verstraete, I. Cirac, cond-mat/0407066

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (as in DMRG)

$$|\Psi\rangle \xrightarrow{\min_{A^1}} |\Psi^1\rangle$$

$$E \quad \geq \quad E^1$$

Variational optimization (e.g. finite PEPS)

e.g. F. Verstraete, I. Cirac, cond-mat/0407066

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (as in DMRG)

$$\begin{array}{ccccccccc}
|\Psi\rangle & \xrightarrow{\min_{A^1}} & |\Psi^1\rangle & \xrightarrow{\min_{A^2}} & \ldots & \xrightarrow{\min_{A^N}} & |\Psi^N\rangle \\
E & \geq & E^1 & \geq & \ldots & \geq & E^N
\end{array}$$

Variational optimization (e.g. finite PEPS)

e.g. F. Verstraete, I. Cirac, cond-mat/0407066

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (as in DMRG)

$$|\Psi\rangle \xrightarrow{\min\limits_{A^1}} |\Psi^1\rangle \xrightarrow{\min\limits_{A^2}} \ldots \xrightarrow{\min\limits_{A^N}} |\Psi^N\rangle \xrightarrow{\min\limits_{A^{N-1}}} |\Psi^{N+1}\rangle$$

$$E \geq E^1 \geq \ldots \geq E^N \geq E^{N+1}$$

Variational optimization (e.g. finite PEPS)

e.g. F. Verstraete, I. Cirac, cond-mat/0407066

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (as in DMRG)

$$|\Psi\rangle \xrightarrow{\min_{A^1}} |\Psi^1\rangle \xrightarrow{\min_{A^2}} \cdots \xrightarrow{\min_{A^N}} |\Psi^N\rangle \xrightarrow{\min_{A^{N-1}}} |\Psi^{N+1}\rangle \xrightarrow{\min_{A^{N-2}}} \cdots$$

$$E \geq E^1 \geq \cdots \geq E^N \geq E^{N+1} \geq \cdots$$

Variational optimization (e.g. finite PEPS)

e.g. F. Verstraete, I. Cirac, cond-mat/0407066

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (as in DMRG)

$$|\Psi\rangle \xrightarrow{\min_{A^1}} |\Psi^1\rangle \xrightarrow{\min_{A^2}} \cdots \xrightarrow{\min_{A^N}} |\Psi^N\rangle \xrightarrow{\min_{A^{N-1}}} |\Psi^{N+1}\rangle \xrightarrow{\min_{A^{N-2}}} \cdots$$

$$E \geq E^1 \geq \cdots \geq E^N \geq E^{N+1} \geq \cdots$$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0$$

A^{*i}

A^{i}

Minimization of quadratic function

Variational optimization (e.g. finite PEPS)

e.g. F. Verstraete, I. Cirac, cond-mat/0407066

$$\min\left(\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)$$

Optimize over each tensor individually and sweep over the entire system (as in DMRG)

$$|\Psi\rangle \xrightarrow{\min_{A^1}} |\Psi^1\rangle \xrightarrow{\min_{A^2}} \cdots \xrightarrow{\min_{A^N}} |\Psi^N\rangle \xrightarrow{\min_{A^{N-1}}} |\Psi^{N+1}\rangle \xrightarrow{\min_{A^{N-2}}} \cdots$$

$$E \geq E^1 \geq \cdots \geq E^N \geq E^{N+1} \geq \cdots$$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0$$

Minimization of quadratic function

$$\mathbf{H}^i_{eff}\vec{A}^i = \lambda \mathbf{N}^i \vec{A}^i$$

Generalized eigenvalue problem

Once $\mathbf{H}^i_{eff}$ and $\mathbf{N}^i$ are known, we can solve this problem efficiently

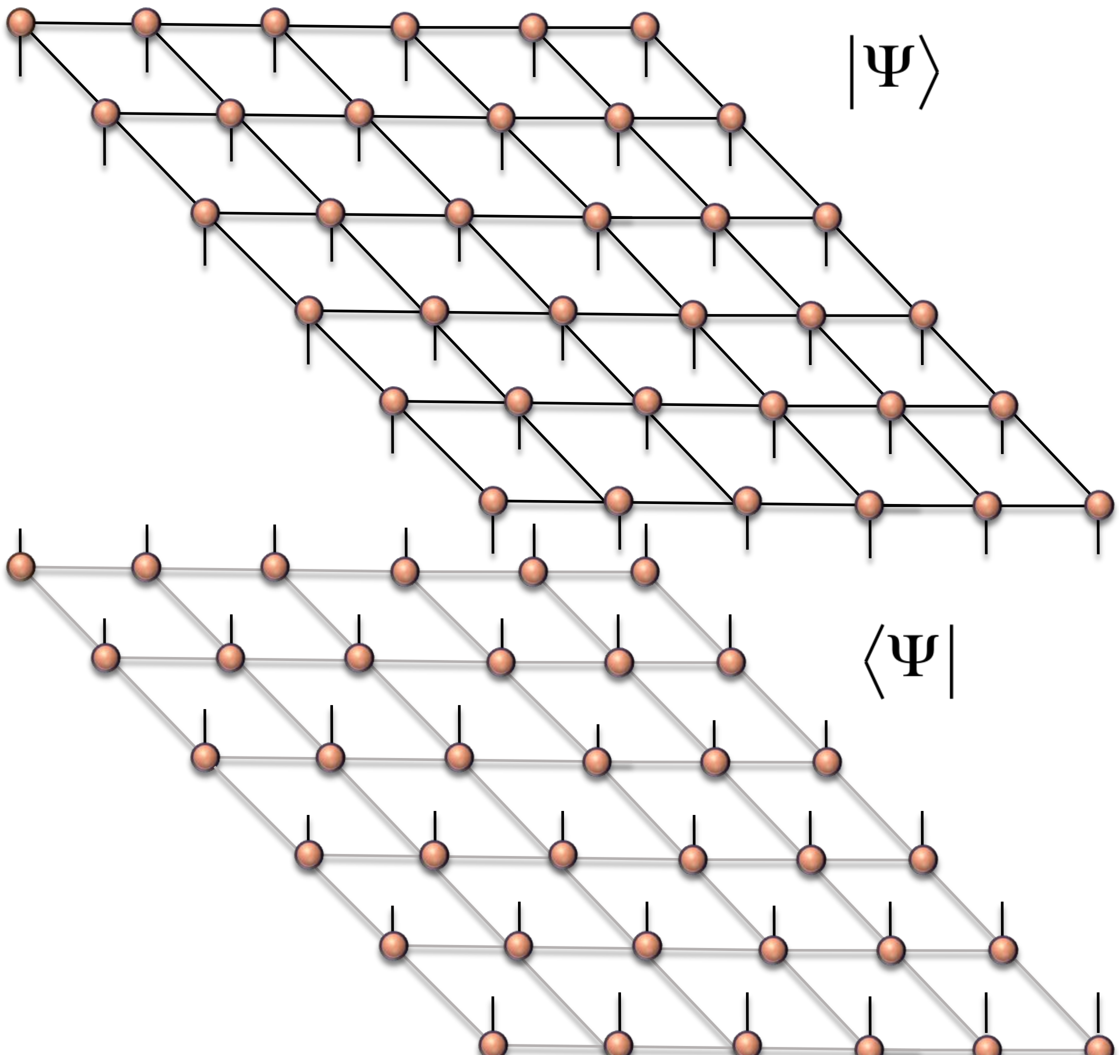
Approximate calculation of $\mathbf{H}^i_{eff}$ and $\mathbf{N}^i$

e.g. calculation of $\mathbf{N}^i\vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle-\lambda\langle\Psi|\Psi\rangle\right)=0 \quad\Rightarrow\quad \mathbf{H}^i_{eff}\vec{A}^i=\lambda\mathbf{N}^i\vec{A}^i$$

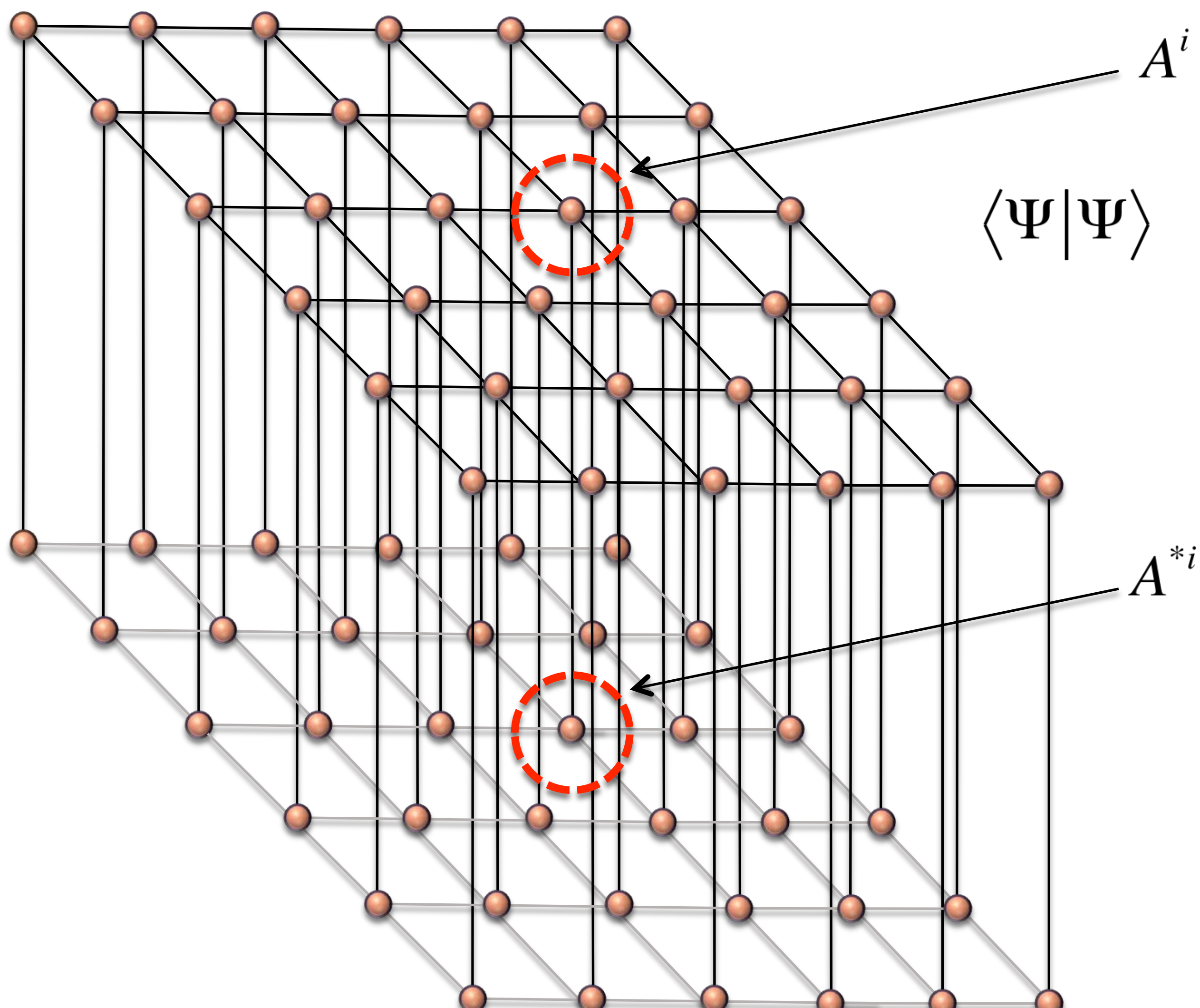
e.g. calculation of $\mathbf{N}^i\vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle-\lambda\langle\Psi|\Psi\rangle\right)=0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i=\lambda\mathbf{N}^i\vec{A}^i$$



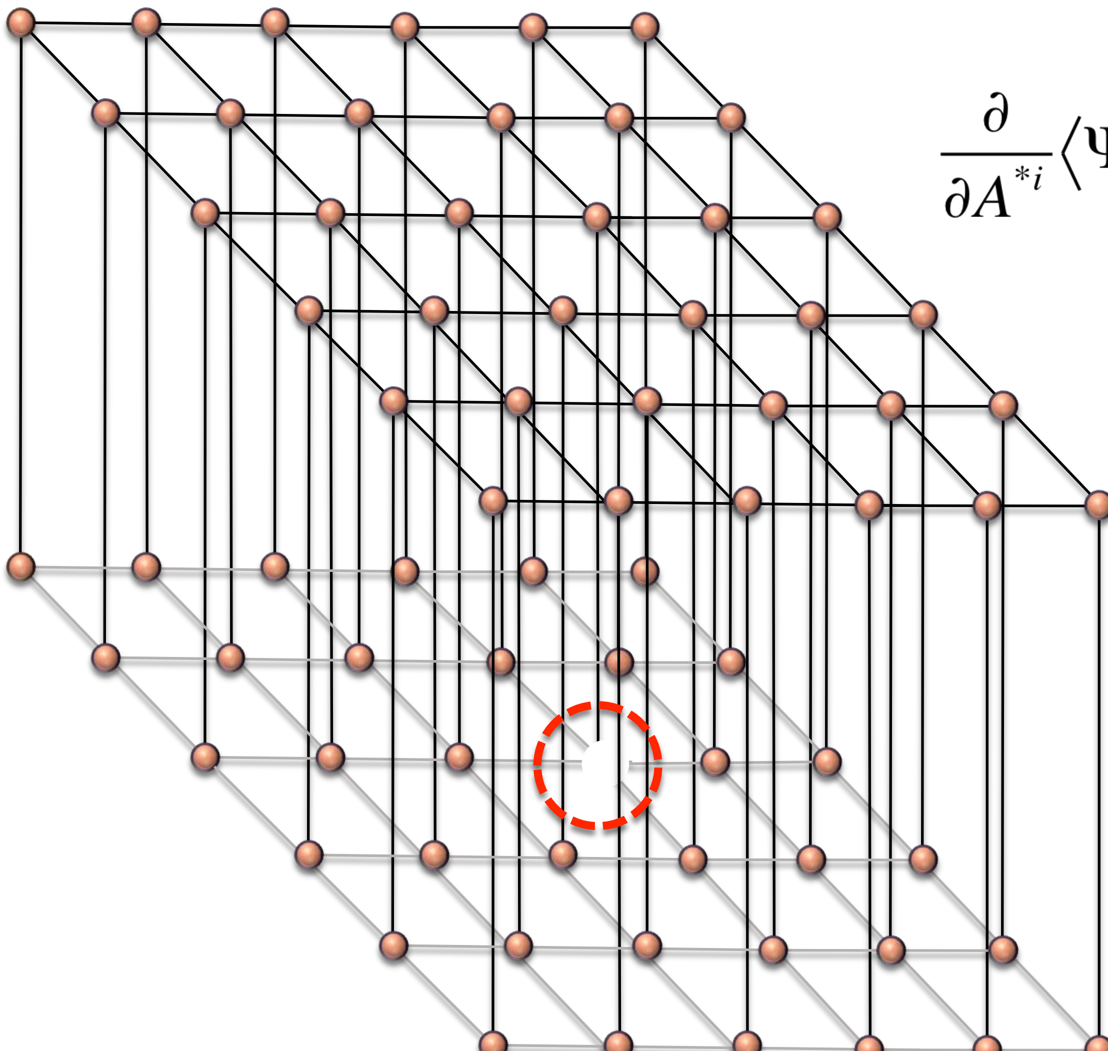
e.g. calculation of $\mathbf{N}^i \vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i = \lambda\mathbf{N}^i\vec{A}^i$$



e.g. calculation of $\mathbf{N}^i \vec{A}^i$

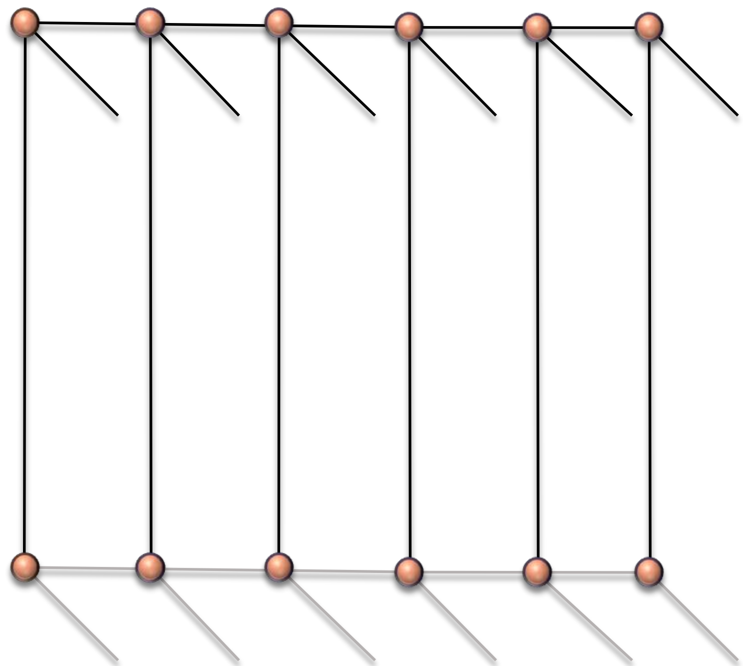
$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i = \lambda\mathbf{N}^i\vec{A}^i$$



$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle = \mathbf{N}^i\vec{A}^i$$

e.g. calculation of $\mathbf{N}^i \vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0 \quad \Rightarrow \quad \mathbf{H}_{eff}^i \vec{A}^i = \lambda \mathbf{N}^i \vec{A}^i$$



$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle = \mathbf{N}^i \vec{A}^i$$

e.g. calculation of $\mathbf{N}^i \vec{A}^i$

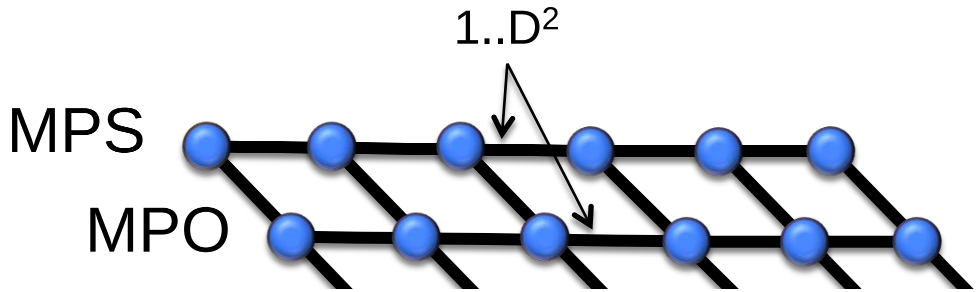
$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i = \lambda \mathbf{N}^i \vec{A}^i$$

MPS

$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle = \mathbf{N}^i \vec{A}^i$$

e.g. calculation of $\mathbf{N}^i \vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i = \lambda\mathbf{N}^i\vec{A}^i$$



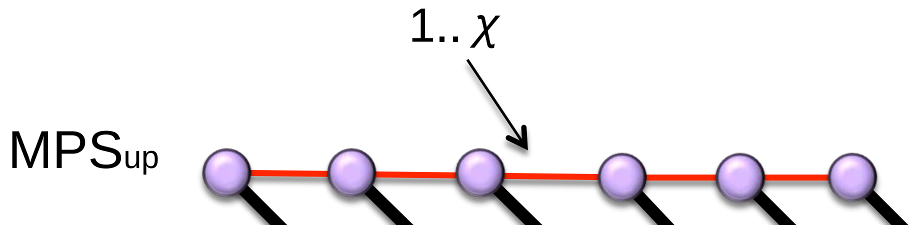
$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle = \mathbf{N}^i\vec{A}^i$$

1d problem: use a 1d method for MPS (e.g., DMRG or TEBD)

e.g. calculation of $\mathbf{N}^i\vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle-\lambda\langle\Psi|\Psi\rangle\right)=0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i=\lambda\mathbf{N}^i\vec{A}^i$$

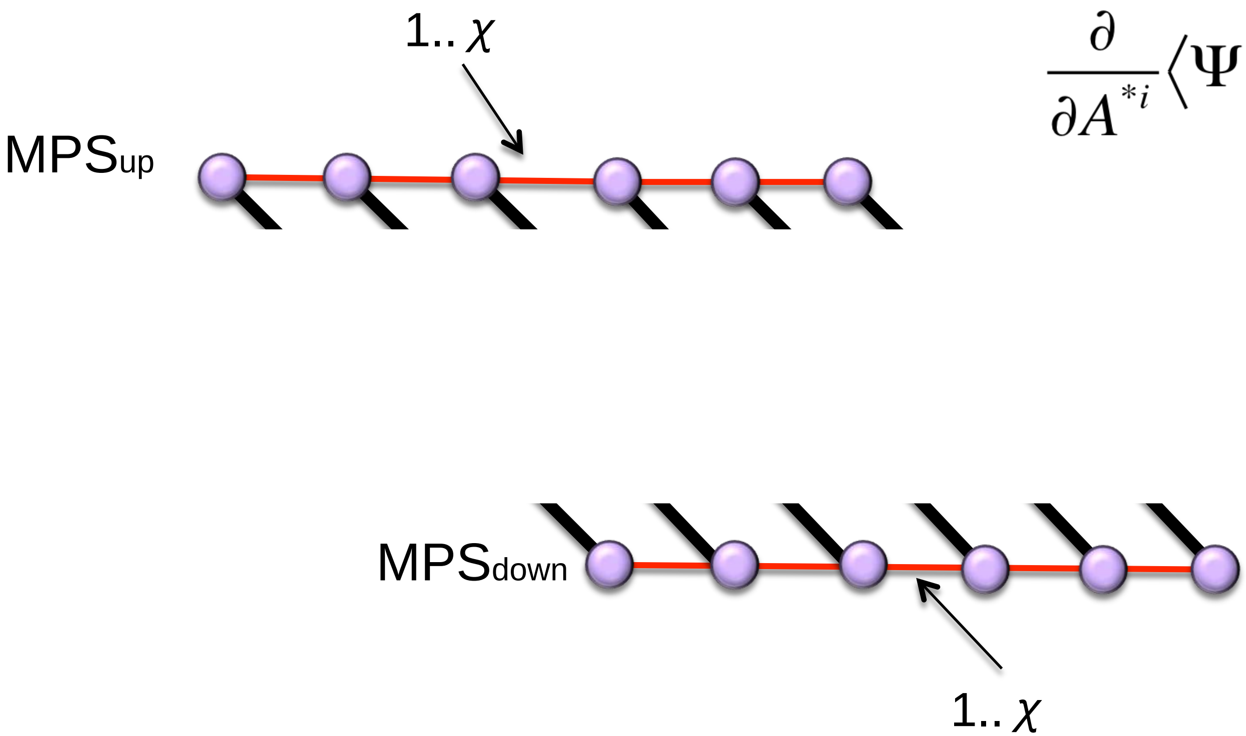
$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle=\mathbf{N}^i\vec{A}^i$$



e.g. calculation of $\mathbf{N}^i\vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle-\lambda\langle\Psi|\Psi\rangle\right)=0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i=\lambda\mathbf{N}^i\vec{A}^i$$

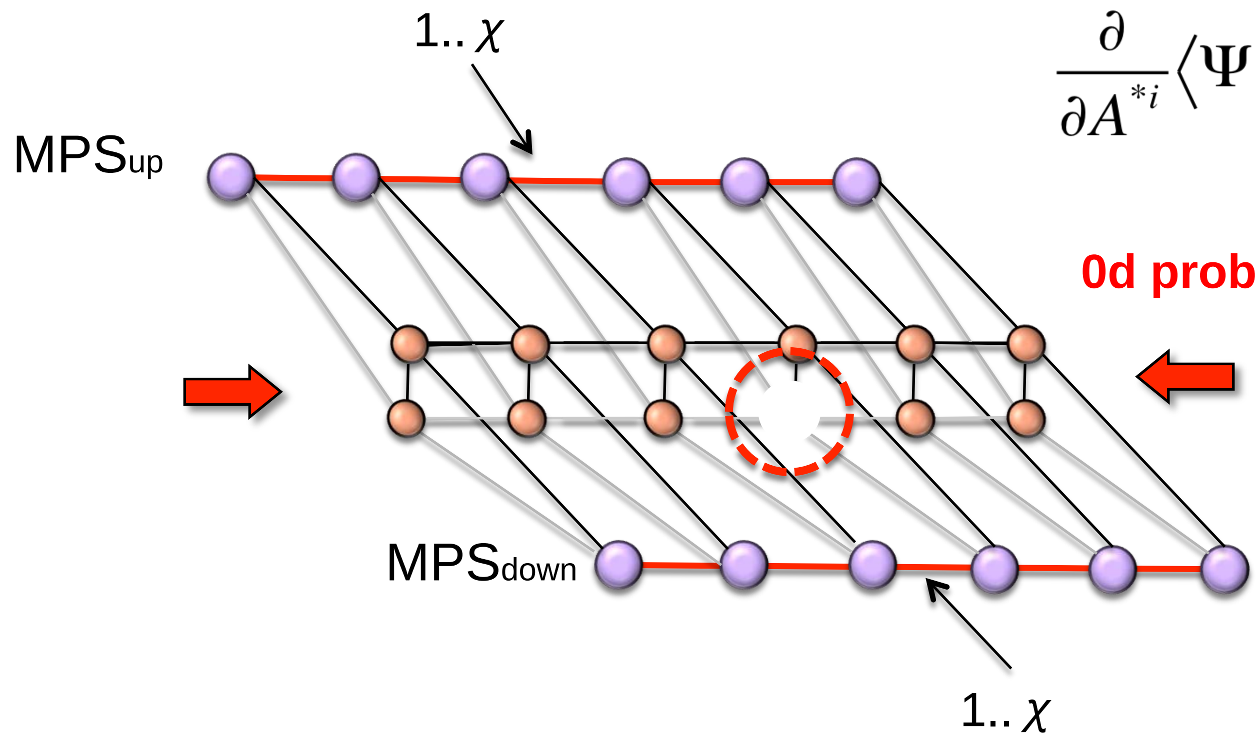
$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle=\mathbf{N}^i\vec{A}^i$$



e.g. calculation of $\mathbf{N}^i\vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i = \lambda\mathbf{N}^i\vec{A}^i$$

$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle = \mathbf{N}^i\vec{A}^i$$



0d problem: exact!

e.g. calculation of $\mathbf{N}^i \vec{A}^i$

$$\frac{\partial}{\partial A^{*i}}\left(\langle\Psi|H|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle\right) = 0 \quad \Rightarrow \quad \mathbf{H}^i_{eff}\vec{A}^i = \lambda \mathbf{N}^i \vec{A}^i$$

$$\frac{\partial}{\partial A^{*i}}\langle\Psi|\Psi\rangle = \mathbf{N}^i \vec{A}^i$$

Dimensional reduction

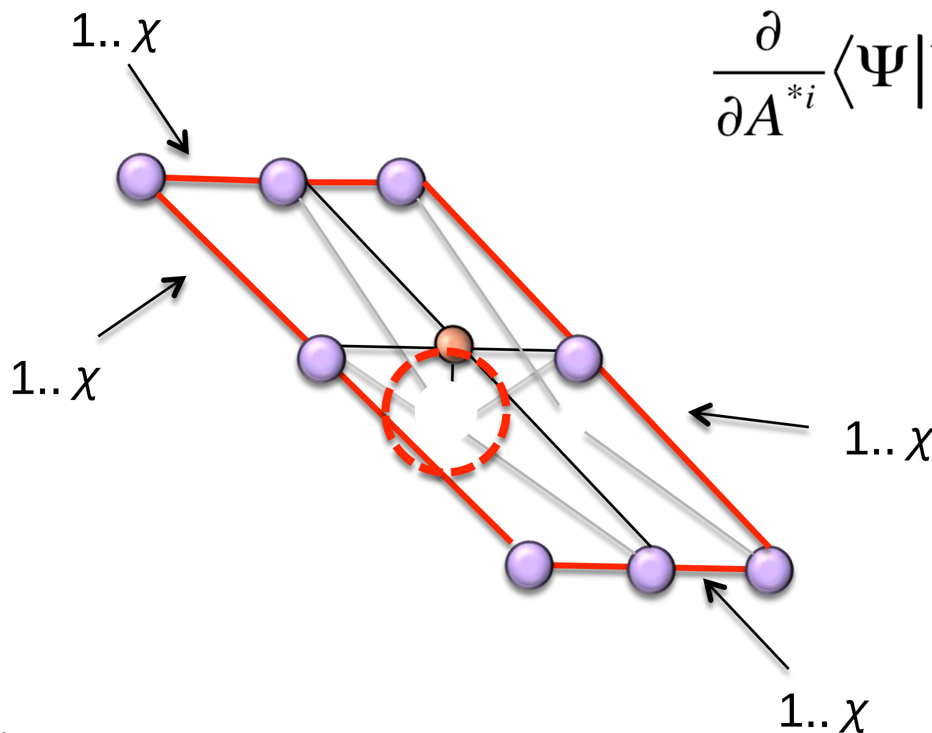
2d problem

↓

1d problem: use DMRG!

↓

0d problem: exact!



$\mathbf{N}^i \vec{A}^i$ is the ***environment*** of tensor A^{*i}

$\mathbf{H}^i_{eff}\vec{A}^i$ is computed similarly, but sandwitching with the Hamiltonian

Valid also for any expectation value

But... does it work?

„Tensor networks provide today the best variational energies for the Hubbard model in the strong coupling limit. iPEPS has really made it“.

Matthias Troyer (at the Korrelationstage 2015)

*P. Corboz, PRB **93** 045116 (2016)*

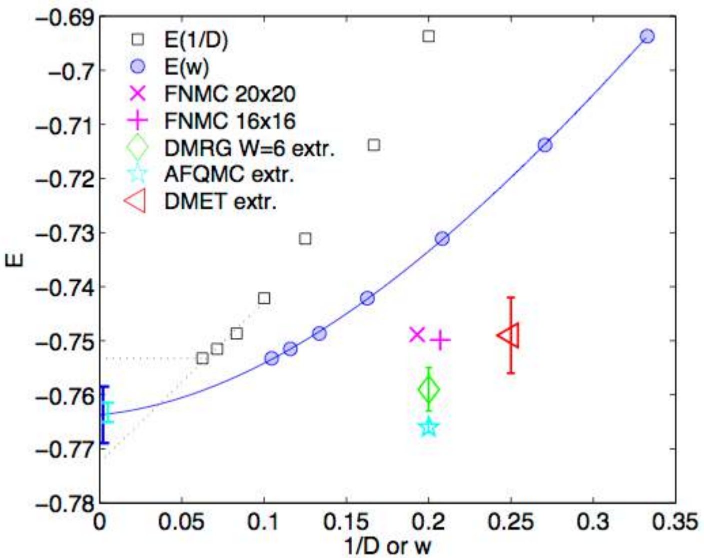
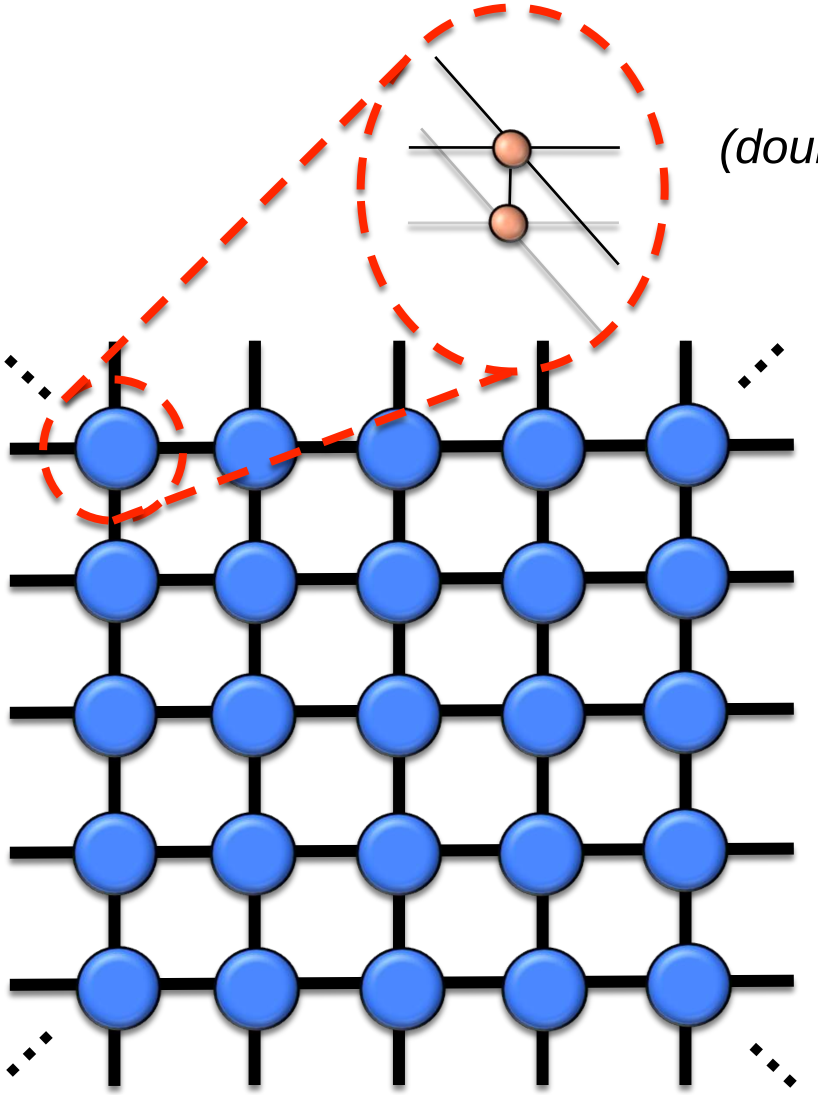


FIG. 4. (Color online) iPEPS energy of a period-5 stripe in the doped case in the strongly correlated regime ($U/t = 8$, $n = 0.875$) in comparison with other methods.

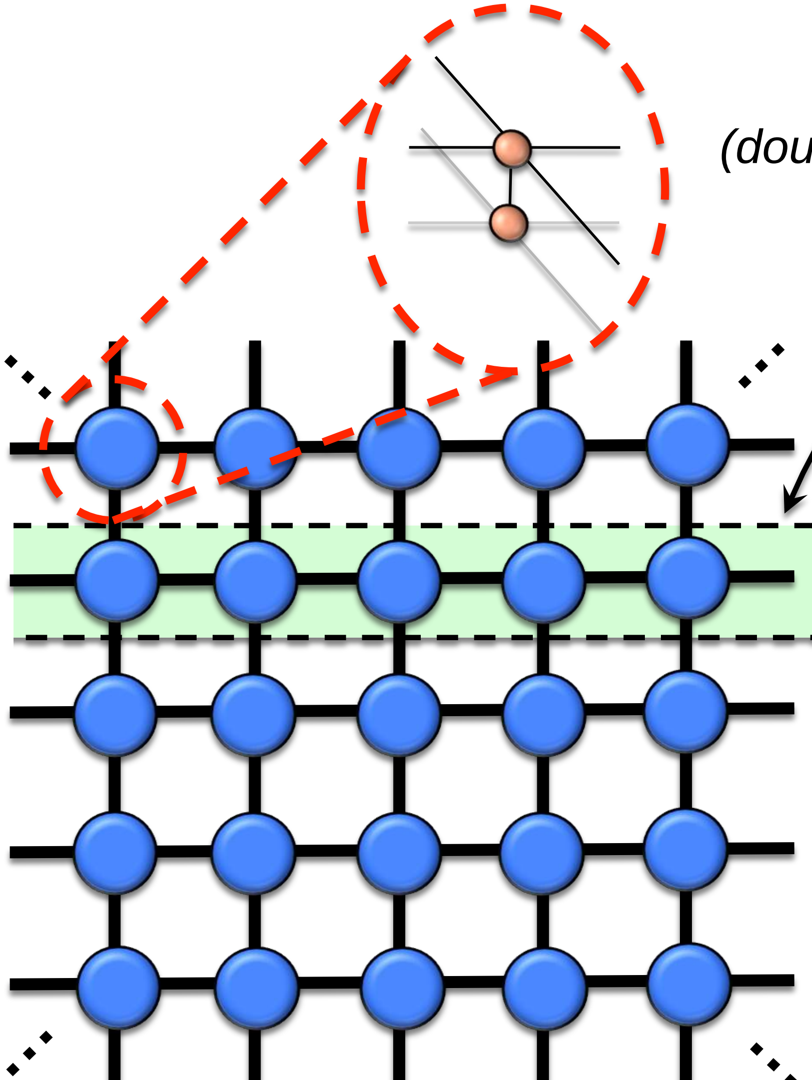
YES, it does

PEPS & Entanglement Hamiltonians

e.g. I. Cirac et al, PRB 83, 245134 (2011), N. Schuch et al, PRL 111, 090501 (2013)



(double indices)

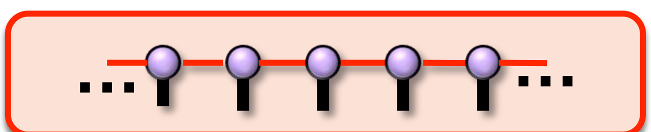


(double indices)

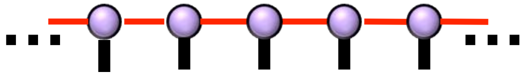
1-dim transfer matrix:

dominant eigenvector?

Can be approximated using infinite MPS



iTEBD, iDMRG, PWFRG, etc

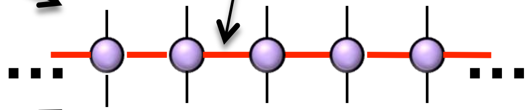


Remember it has double indices…

Virtual indices of bra

1…D

Boundary virtual index 1... χ



Virtual indices of ket

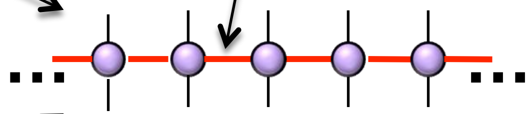
1…D

It is also hermitian and positive by construction (up to finite- χ *effects)*

Virtual indices of bra

1…D

Boundary virtual index 1... χ



Virtual indices of ket

1…D

It is also hermitian and positive by construction (up to finite-χ effects)

$$\|$$

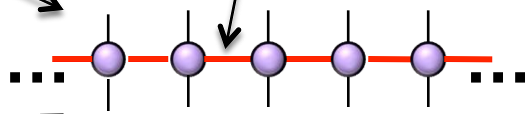
1d Entanglement Hamiltonian

$$\rho = \exp(-H_E)$$

Who is** H_E **???

Virtual indices of bra
1…D

Boundary virtual index 1… χ



Virtual indices of ket
1…D

It is also hermitian and positive by construction (up to finite-χ effects)

$$||$$

1d Entanglement Hamiltonian

$$\rho = \exp(-H_E)$$

Who is H_E ***???***

Bulk	**Holographic principle**	**Boundary**
Gapped 2d systems, trivial phase		***1d Hamiltonian, short-range***
Critical 2d systems	$\Rightarrow$	***1d Hamiltonian, long-range***
Gapped 2d systems, topological order		***Completely non-local (projector)***

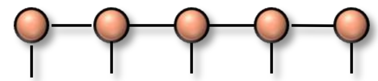
Based on examples

Particles and energies from Hamiltonians,

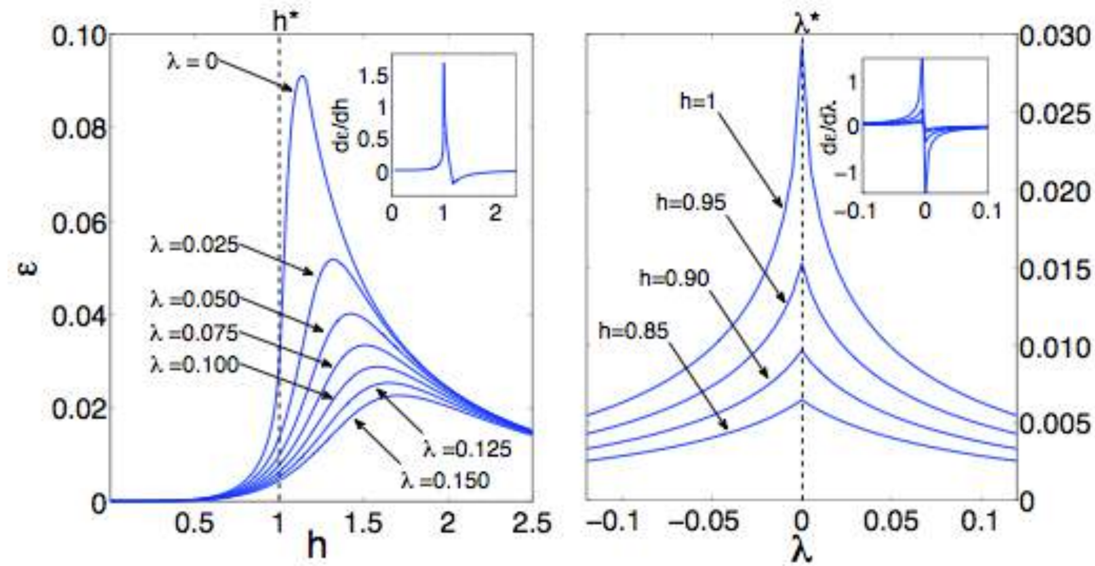
and Hamiltonians from networks of entanglement + holography

7) Multiscale Entanglement Renormalization Ansatz (MERA)

From MPS to MERA

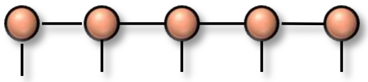
Matrix Product States (MPS)  *1d systems*

But we want to do critical systems!!!

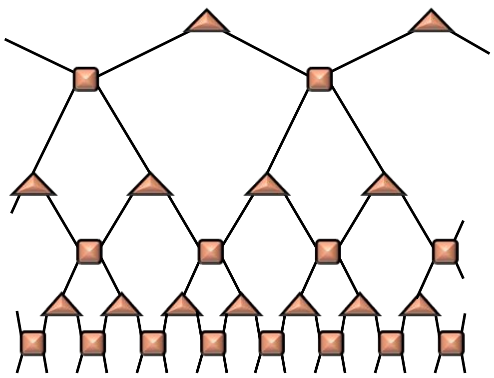


Also very painful for DMRG...

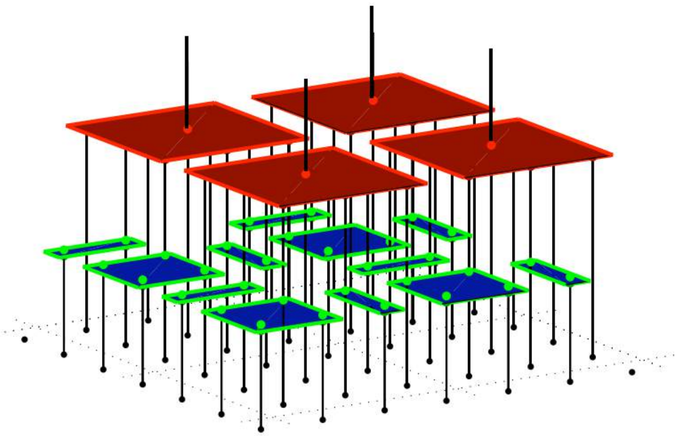
From MPS to MERA

Matrix Product States (MPS)  *1d systems*

Multiscale Entanglement Renormalization Ansatz (MERA)



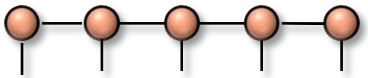
1d systems

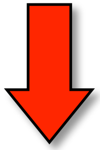


2d systems

and so on…

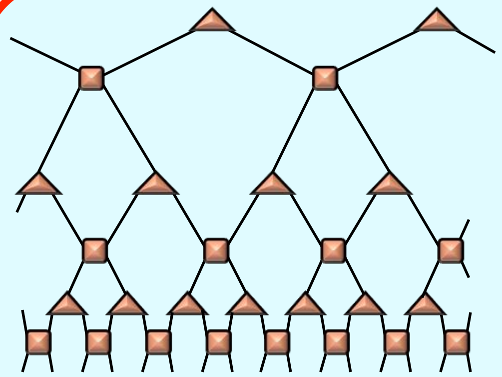
From MPS to MERA

Matrix Product States (MPS)  *1d systems*

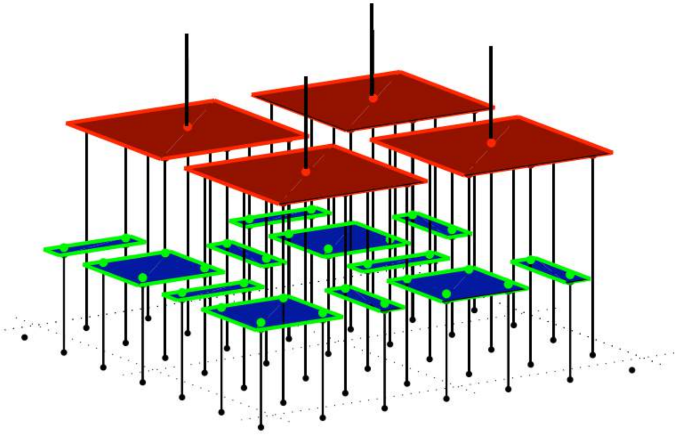


Multiscale Entanglement Renormalization Ansatz (MERA)

This lecture



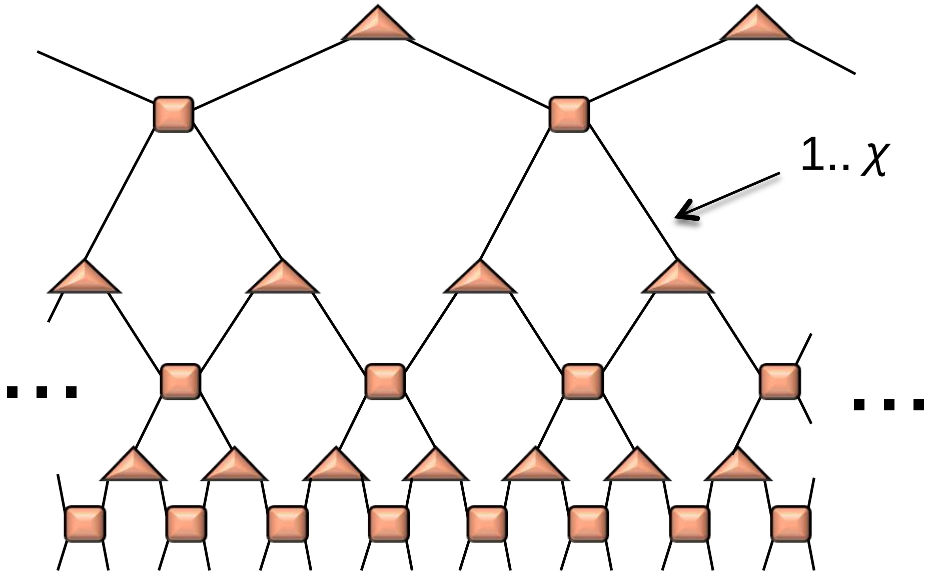
1d systems



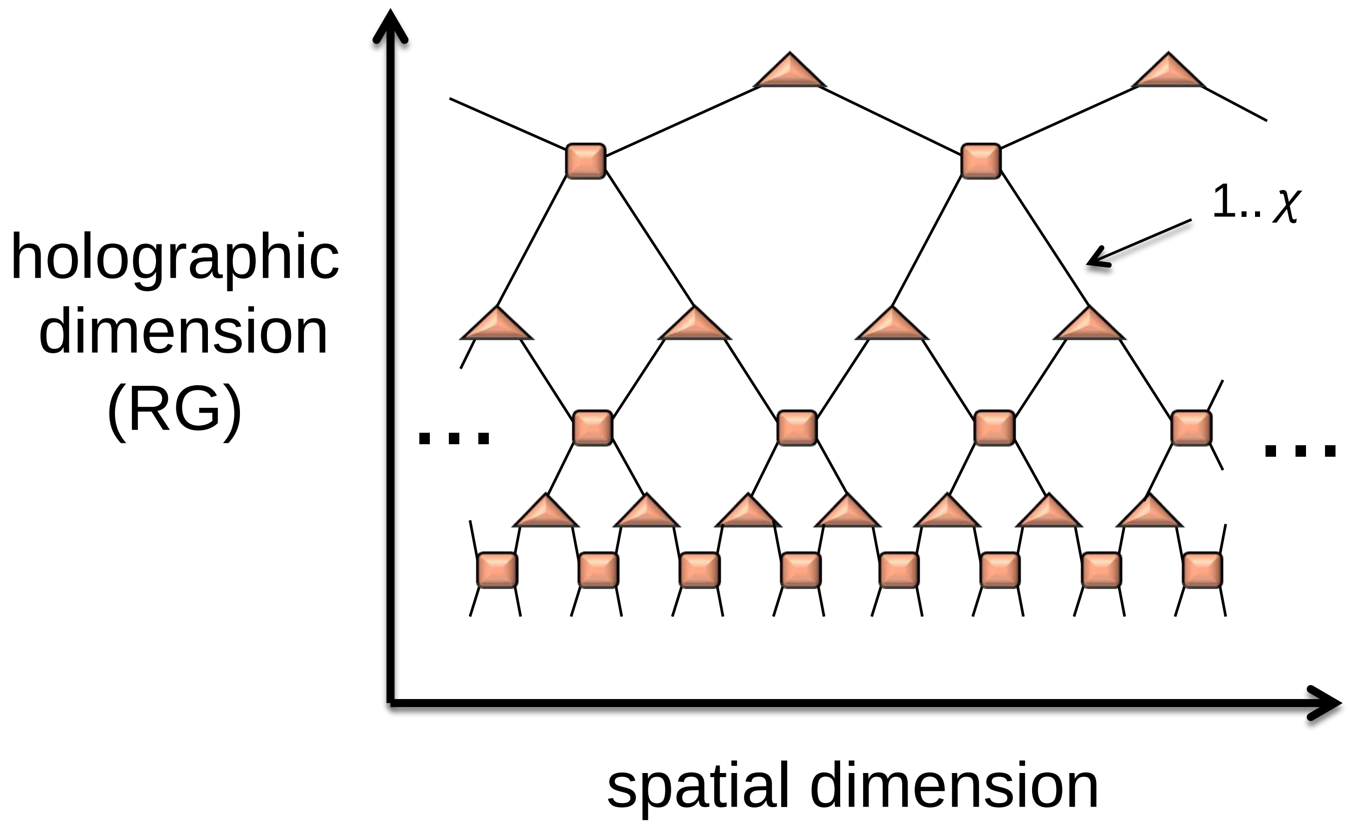
2d systems

and so on…

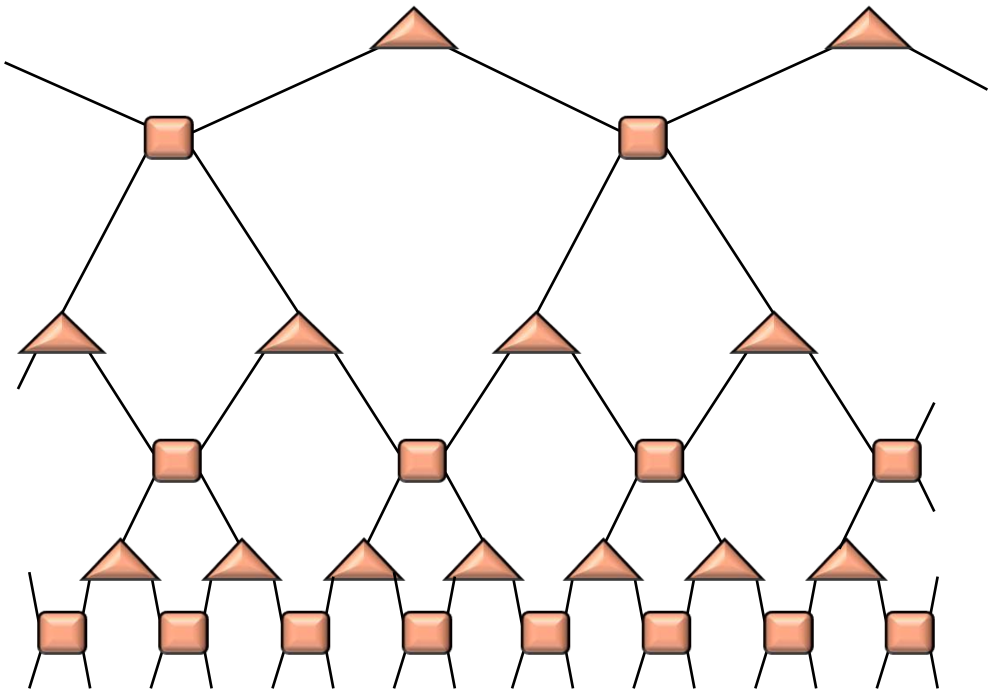
1d MERA

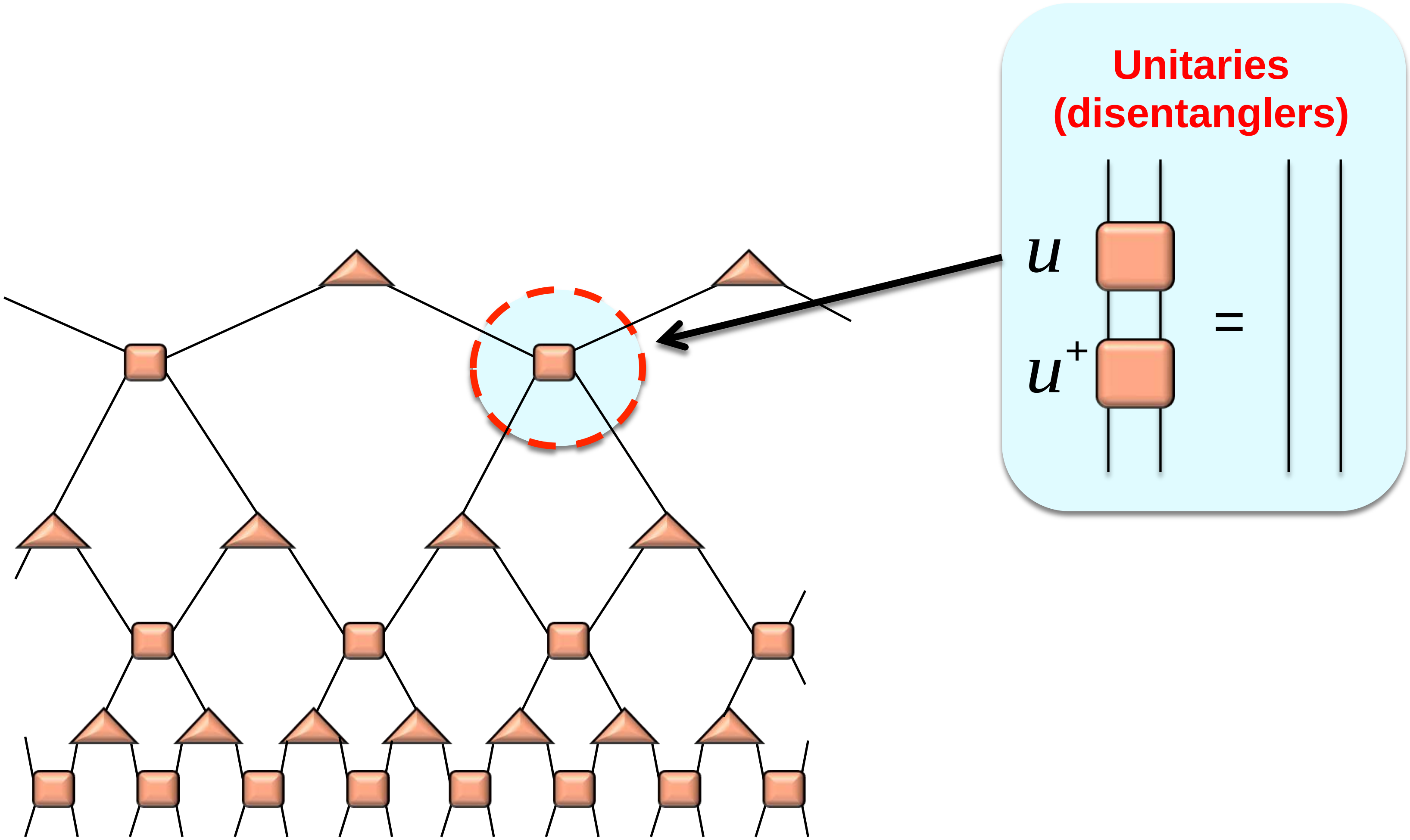


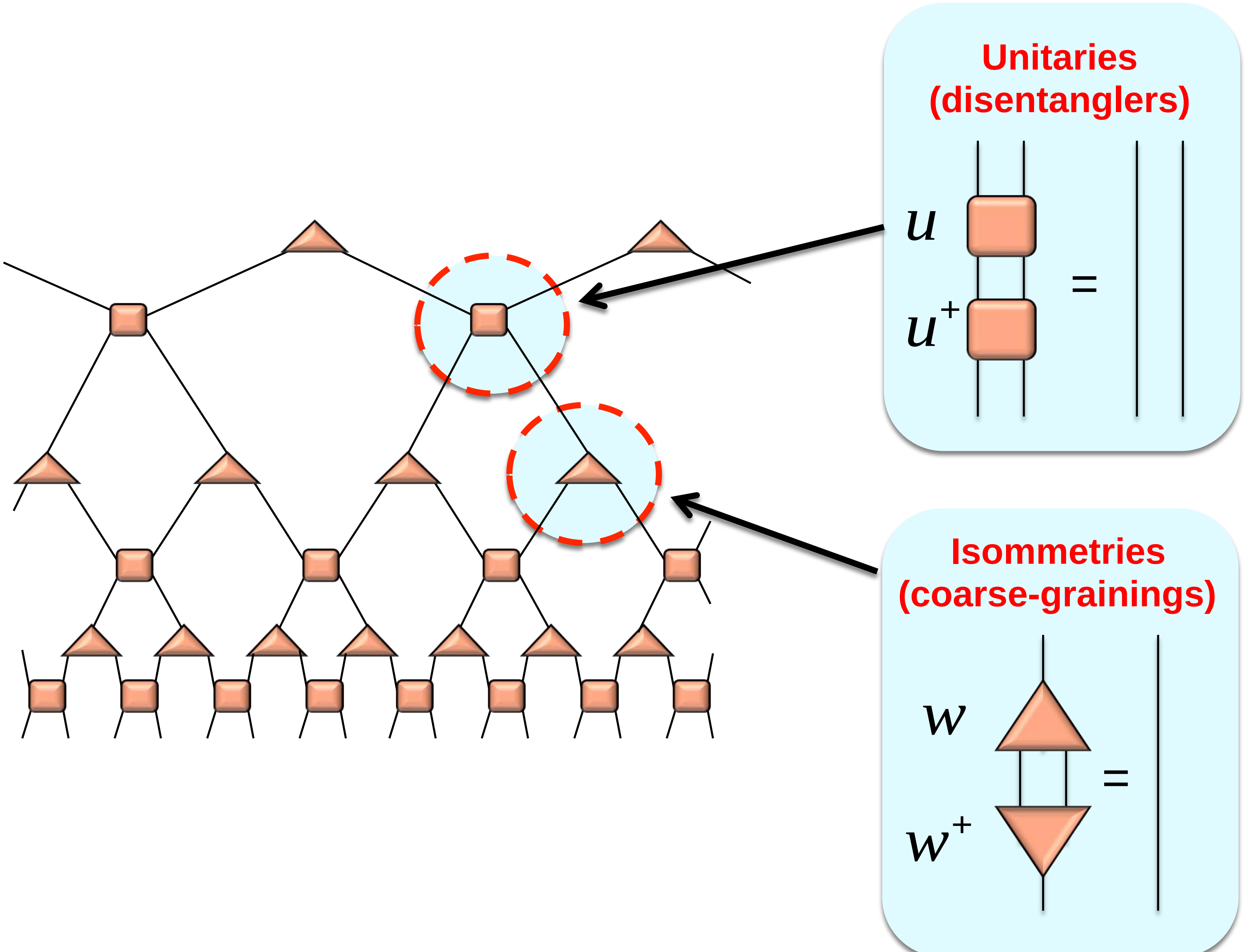
1d MERA



Tensors obey constraints




Unitaries
(disentanglers)
u
u^+
=

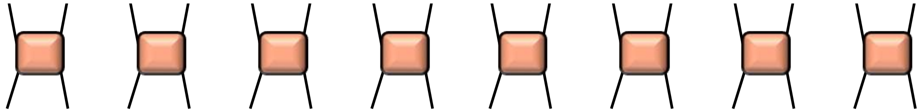


Unitaries
(disentanglers)
u
u+
=
Isommetries
(coarse-grainings)
w
w+
=

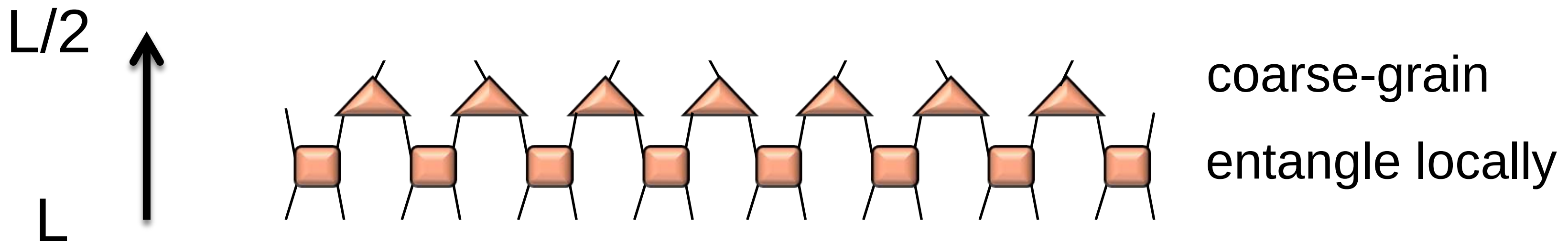
Reason:

entanglement is built locally
at all length scales

L

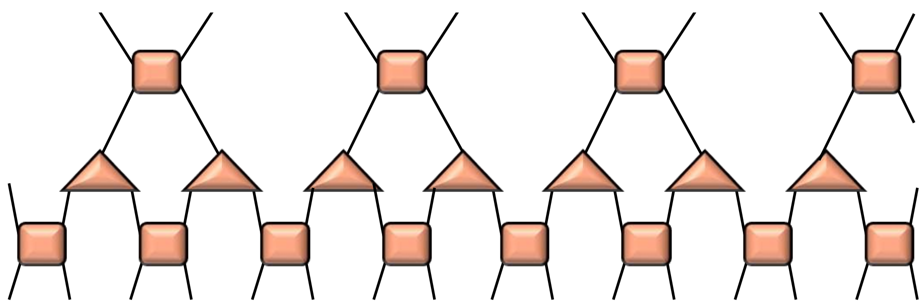


entangle locally


L/2
L
coarse-grain
entangle locally

L/2

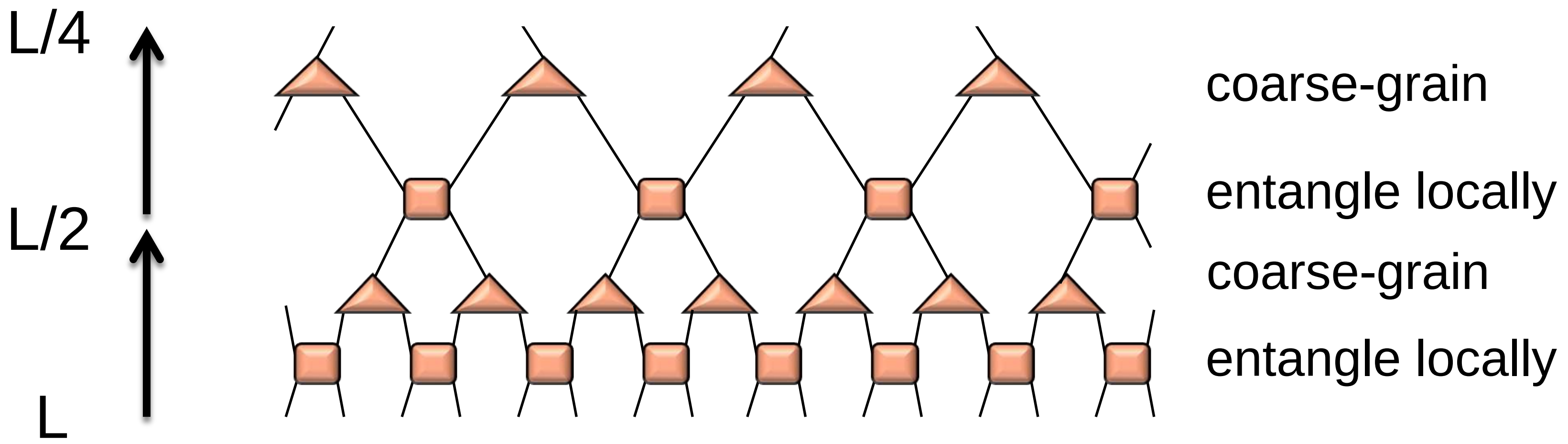
L

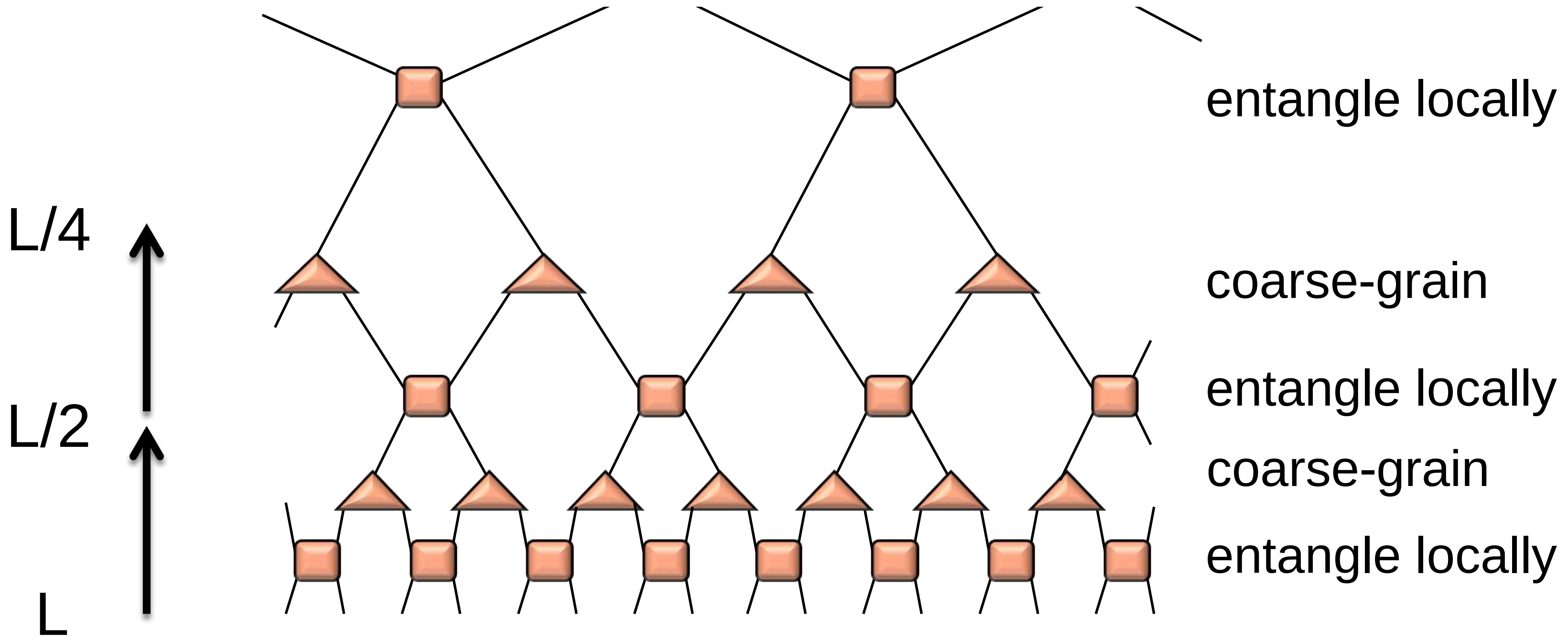


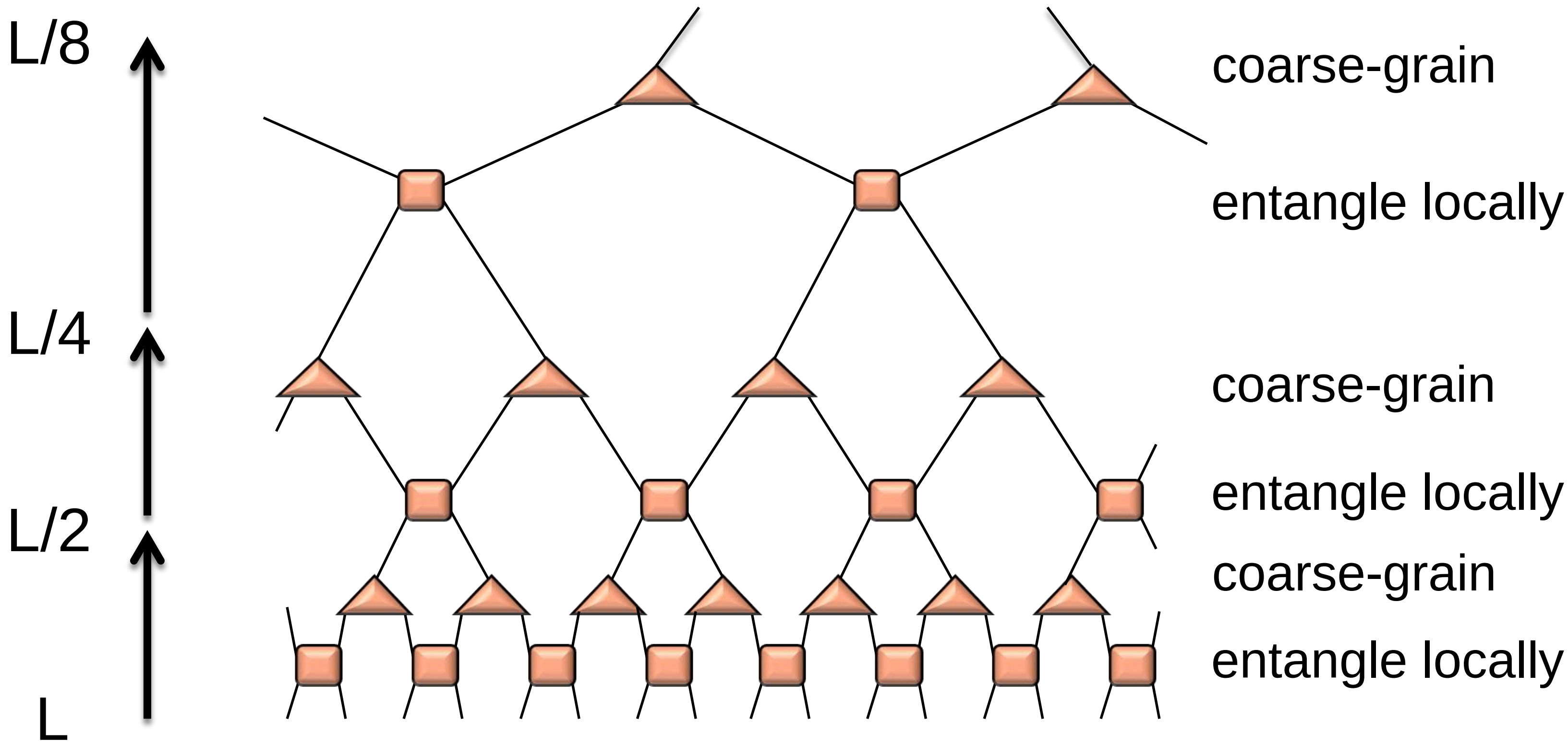
entangle locally

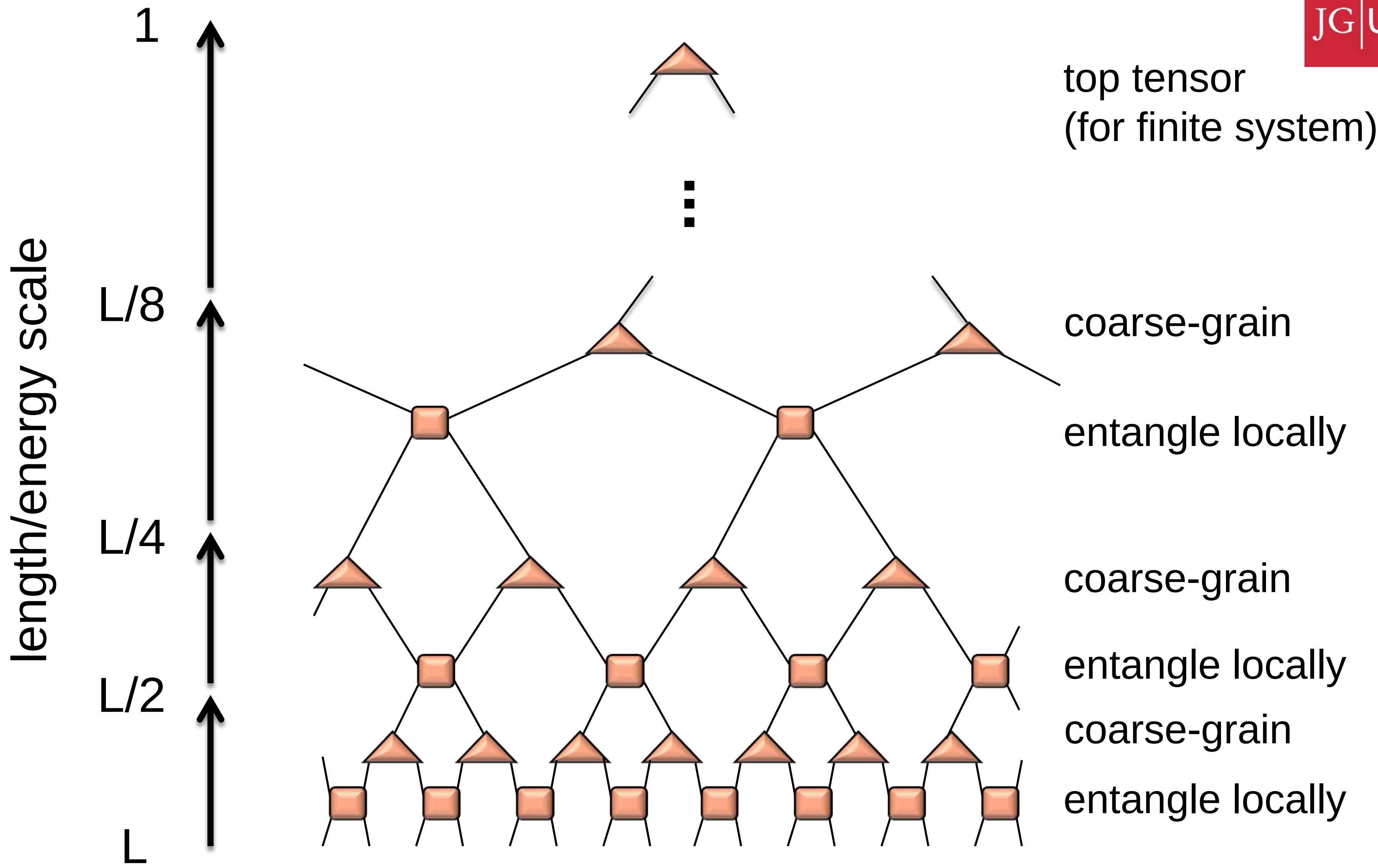
coarse-grain

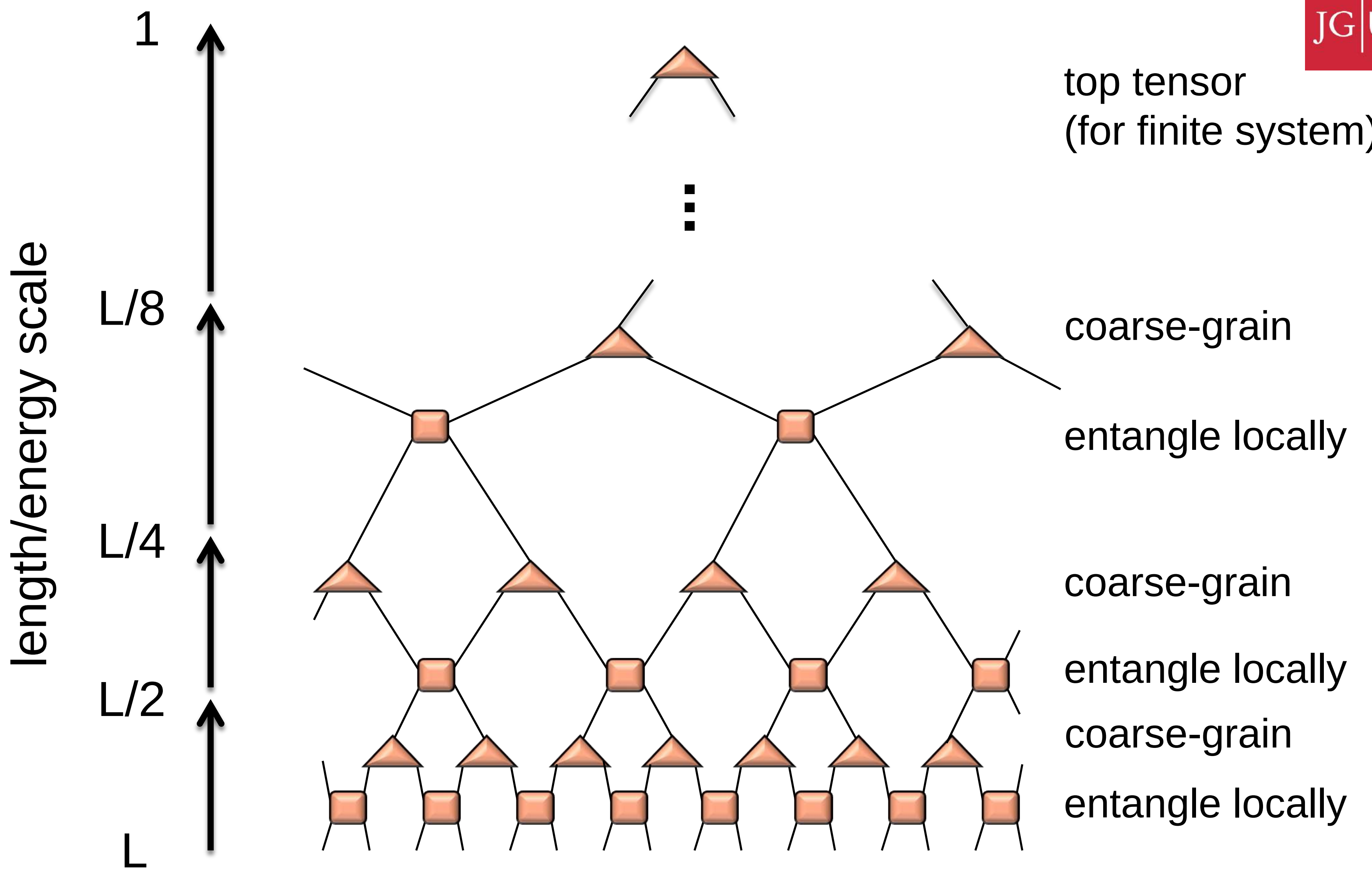
entangle locally


L/4
L/2
L
coarse-grain
entangle locally
coarse-grain
entangle locally


entangle locally
coarse-grain
entangle locally
coarse-grain
entangle locally
L/4
L/2
L

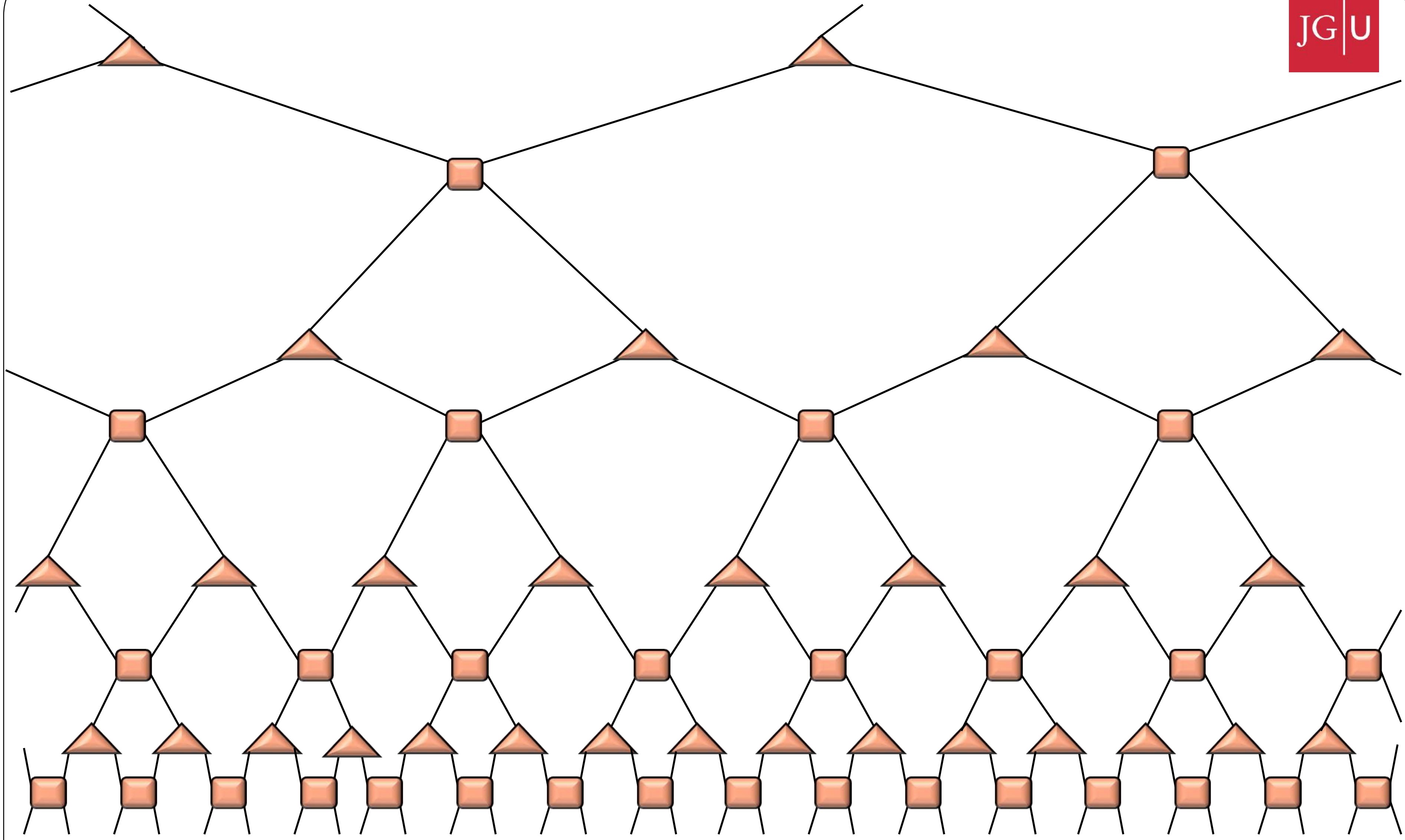

L/8
L/4
L/2
L
coarse-grain
entangle locally
coarse-grain
entangle locally
coarse-grain
entangle locally

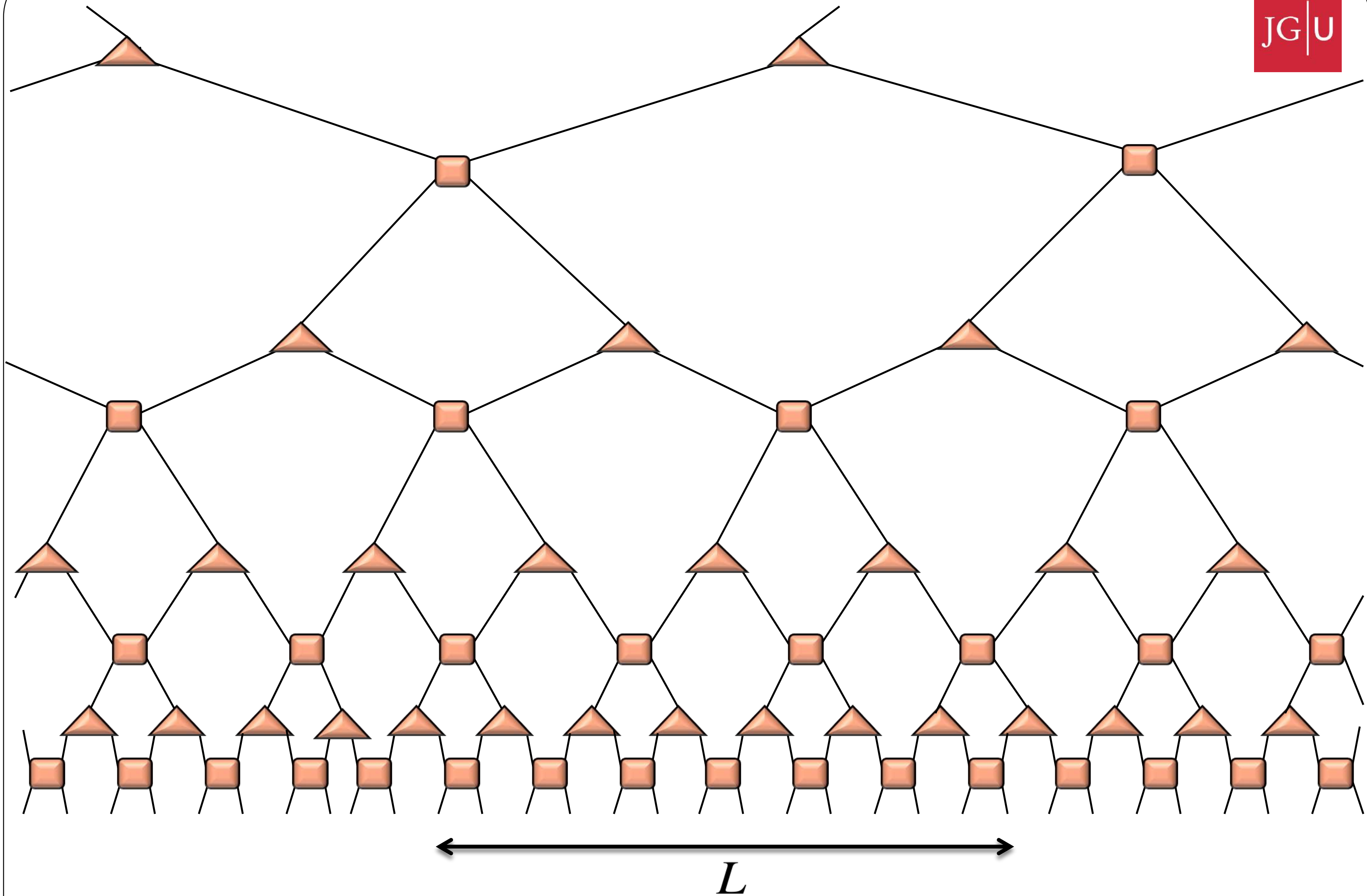

1
L/8
L/4
L/2
L
length/energy scale
top tensor
(for finite system)
coarse-grain
entangle locally
coarse-grain
entangle locally
coarse-grain
entangle locally

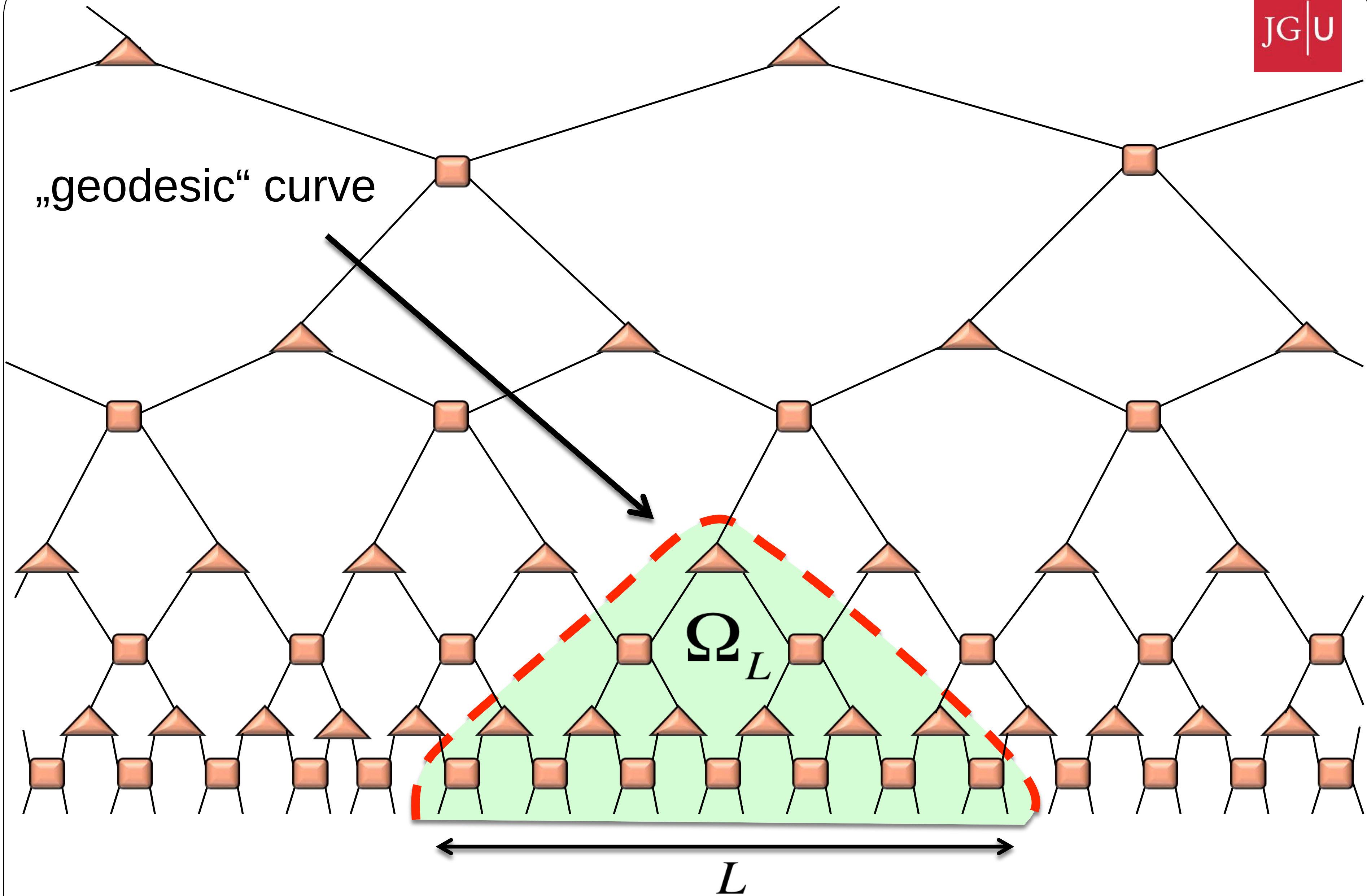


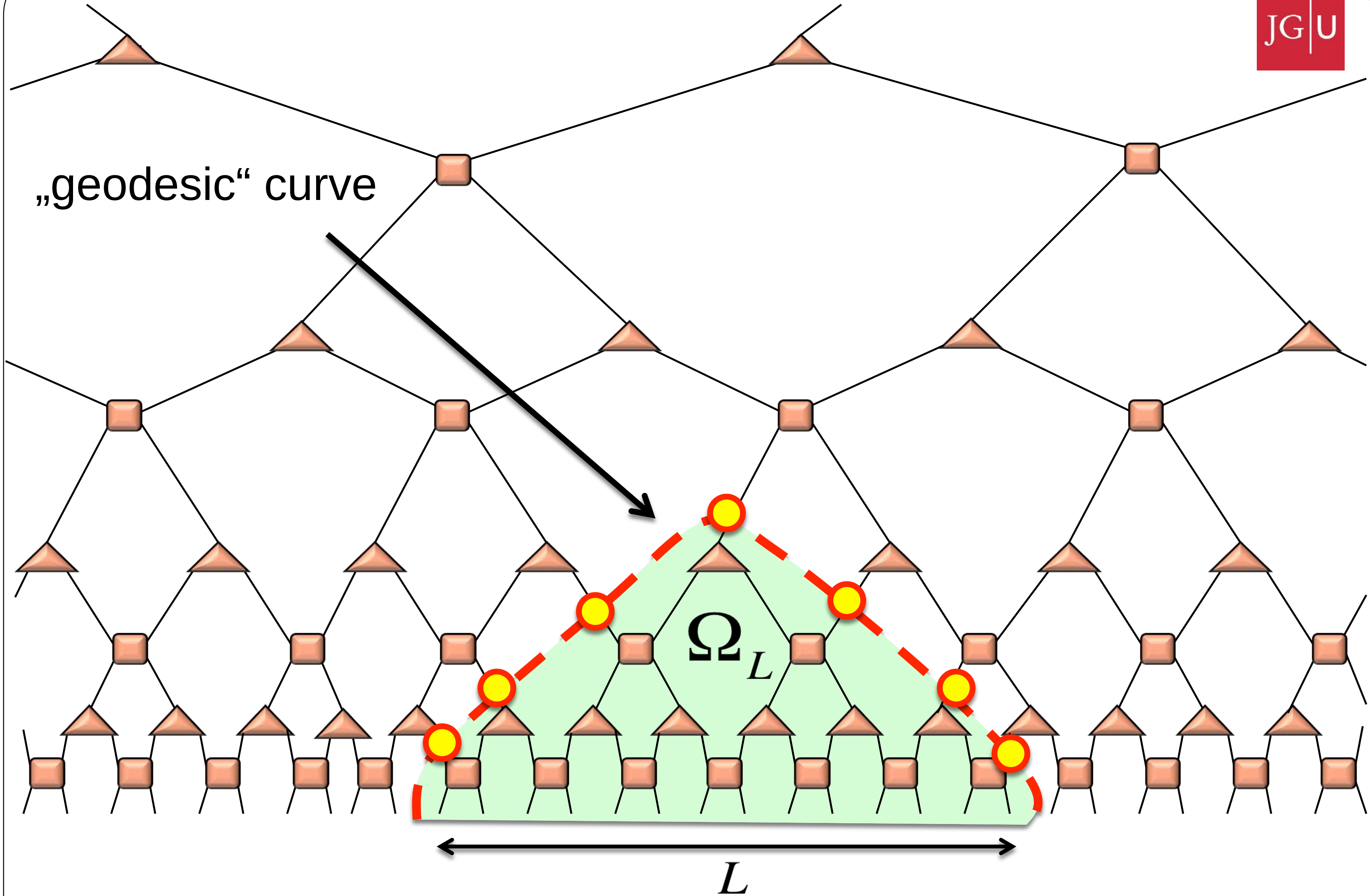
Extra dimension defines an RG flow: Entanglement Renormalization

Entropy of 1d MERA

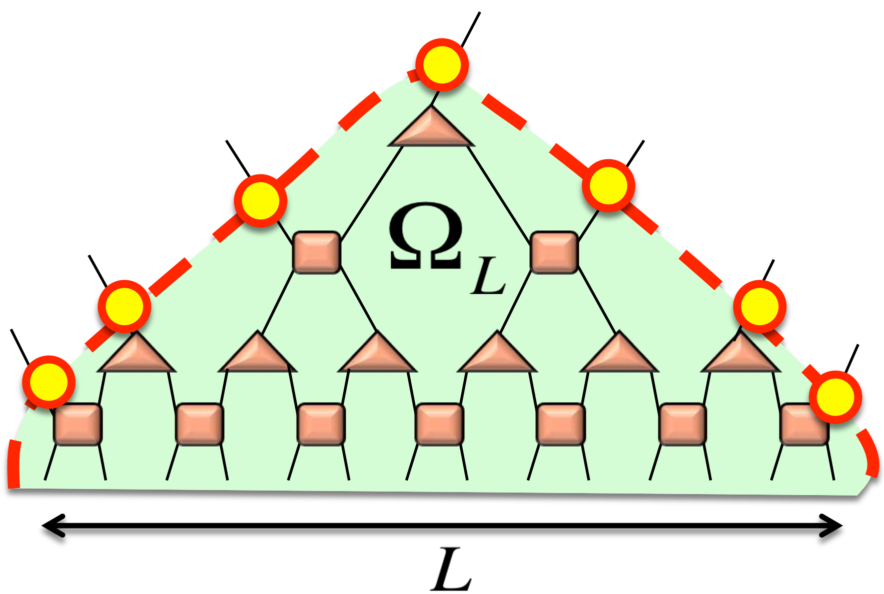



L

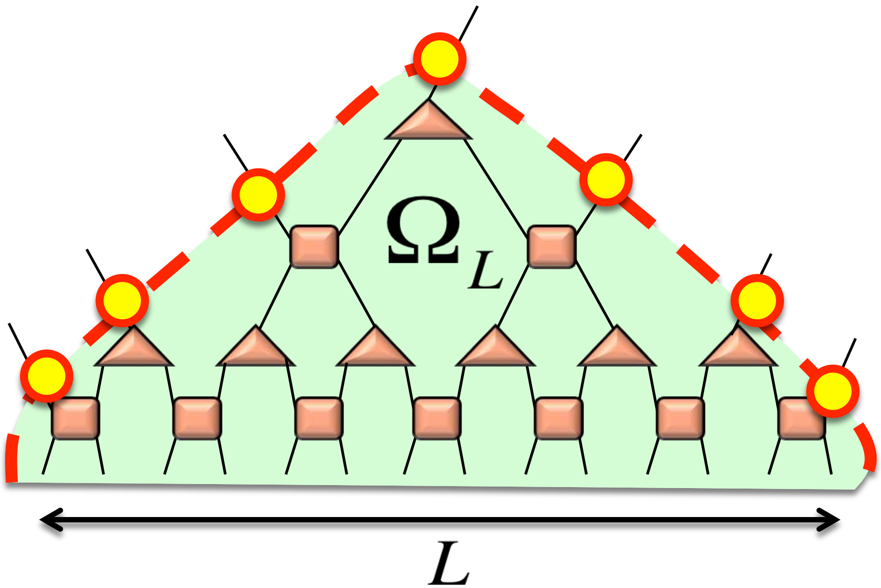

„geodesic“ curve
Ω_L
L



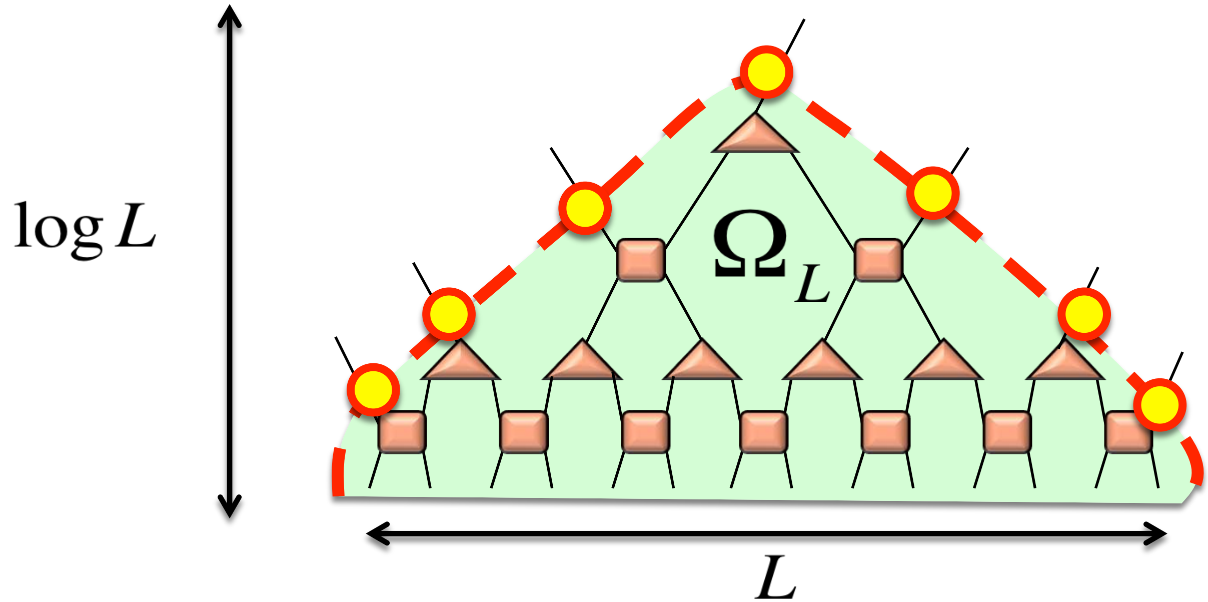
Entanglement as boundary in holographic geometry: $S(L) \leq \log(\chi)\left|\partial\Omega_L\right|$



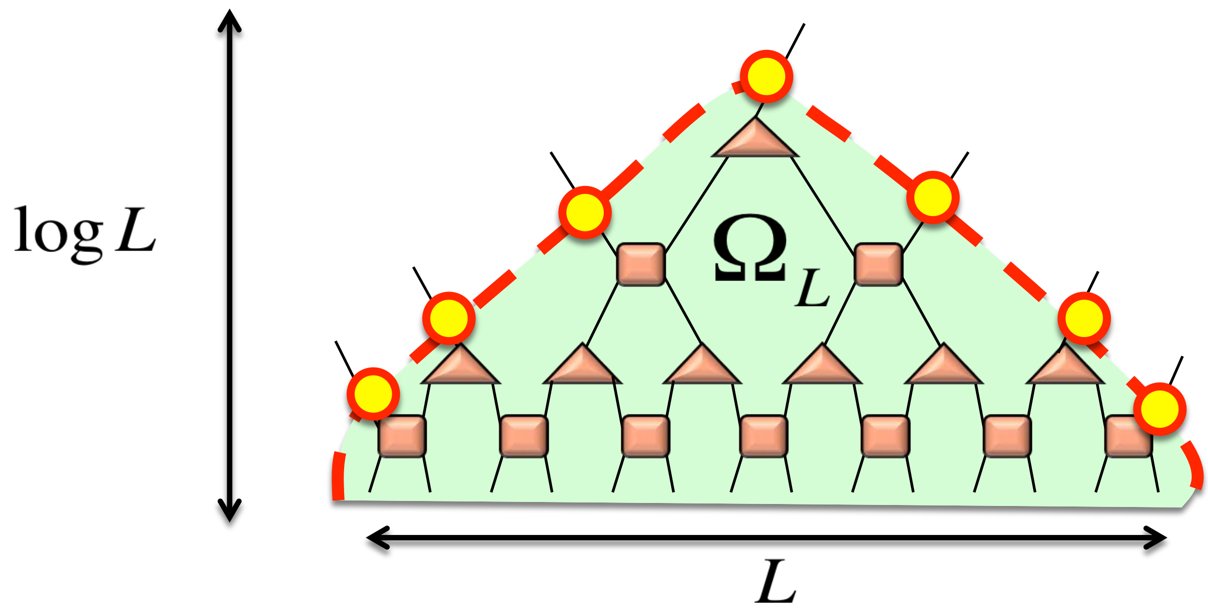
Entanglement as boundary in holographic geometry: $S(L) \le \log(\chi)\left|\partial\Omega_L\right|$



Entanglement as boundary in holographic geometry: $S(L) \leq \log(\chi)\left|\partial\Omega_L\right|$



Entanglement as boundary in holographic geometry: $S(L) \le \log(\chi)\left|\partial\Omega_L\right|$



Entanglement as boundary in holographic geometry: $S(L) \leq \log(\chi)\left|\partial\Omega_L\right|$

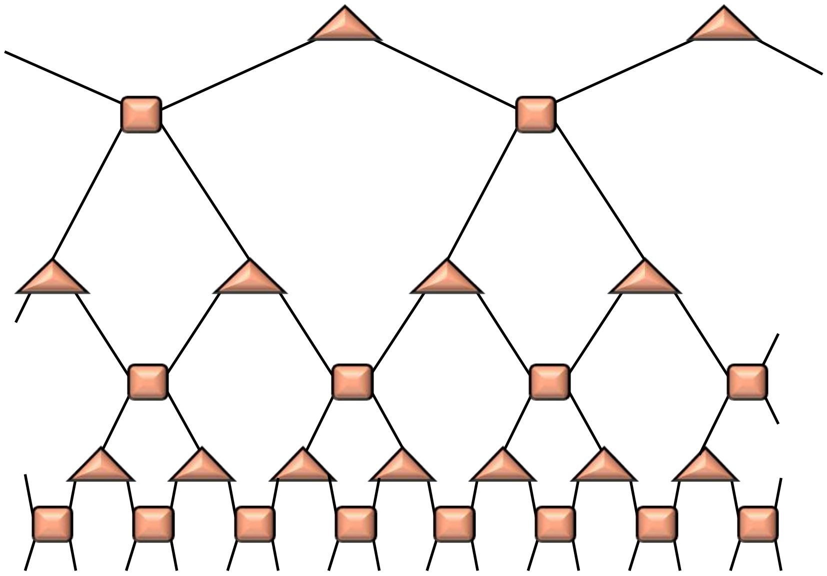
Constant contribution at every layer

1d MERA can produce logarithmic violations to the area-law: $S(L) \approx \log L$

(like 1d critical systems!)

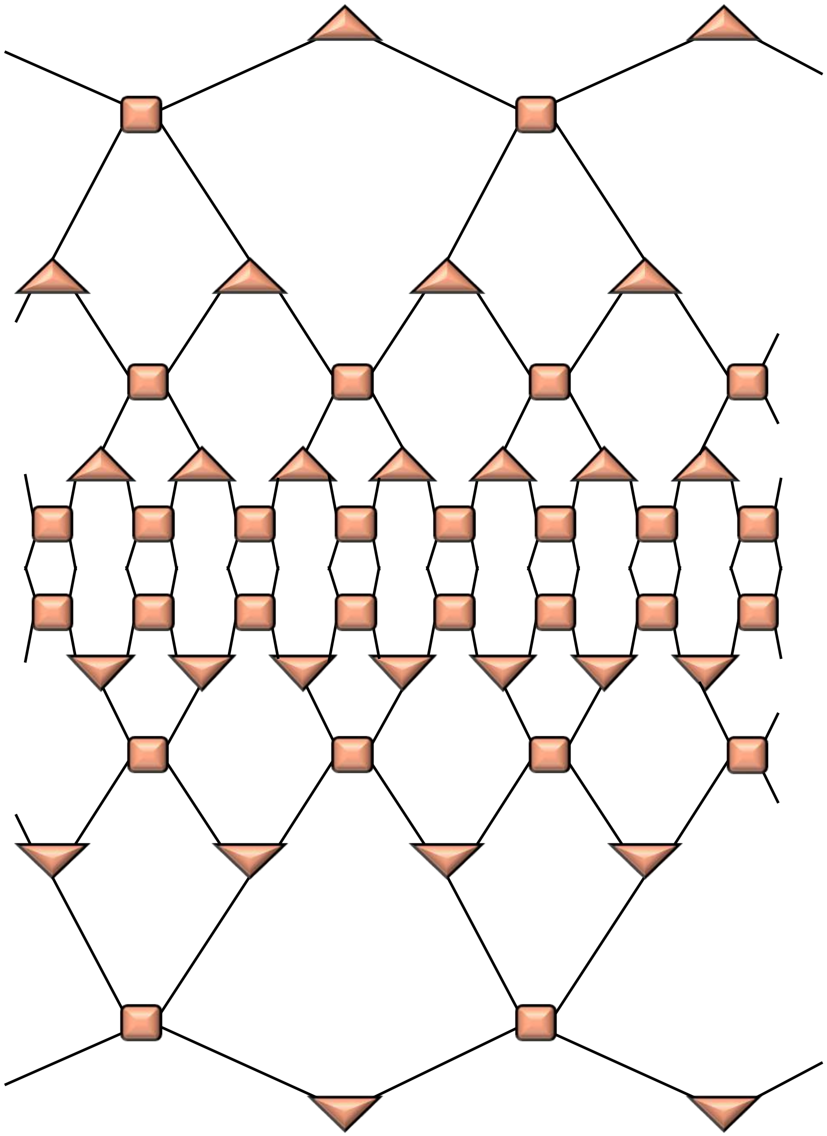
Norm of MERA

Norm of MERA



$$|\Psi\rangle$$

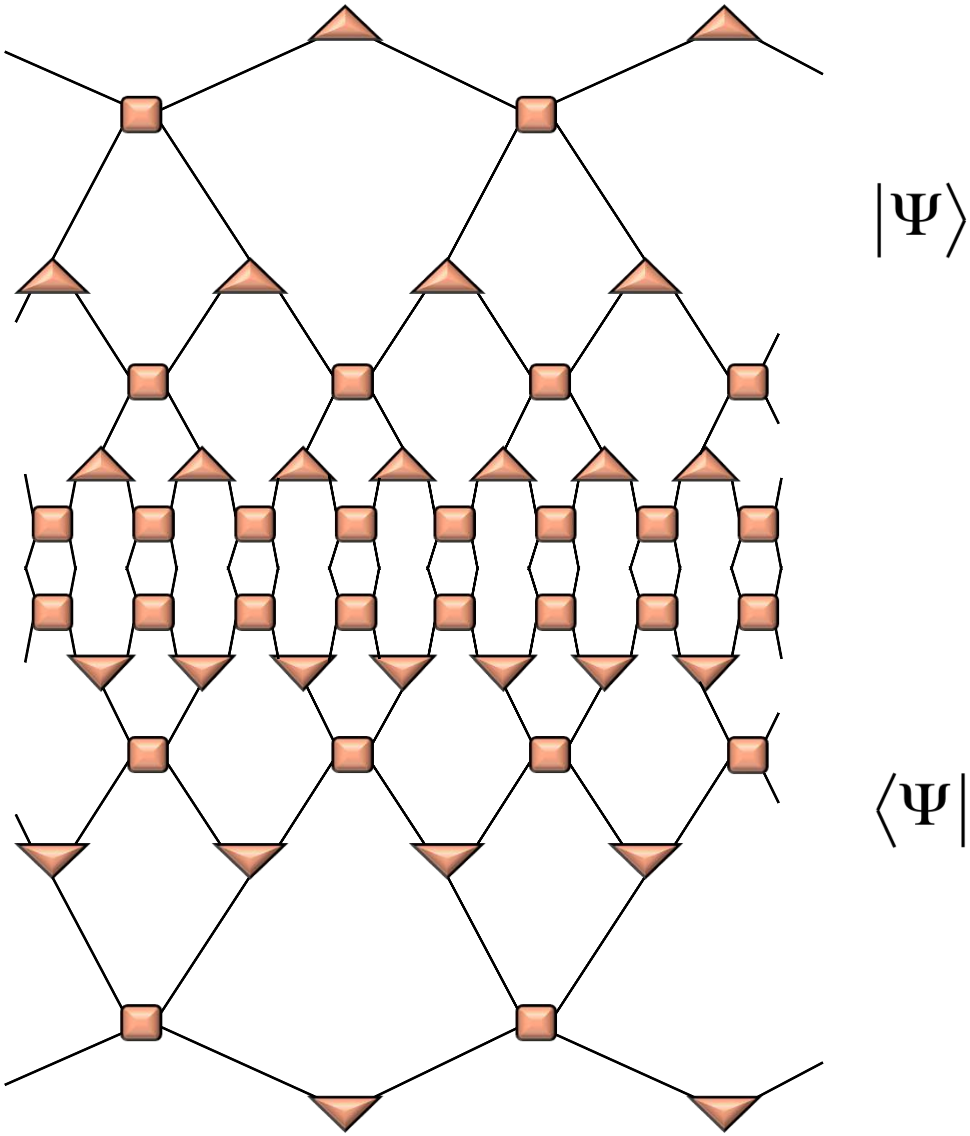
Norm of MERA

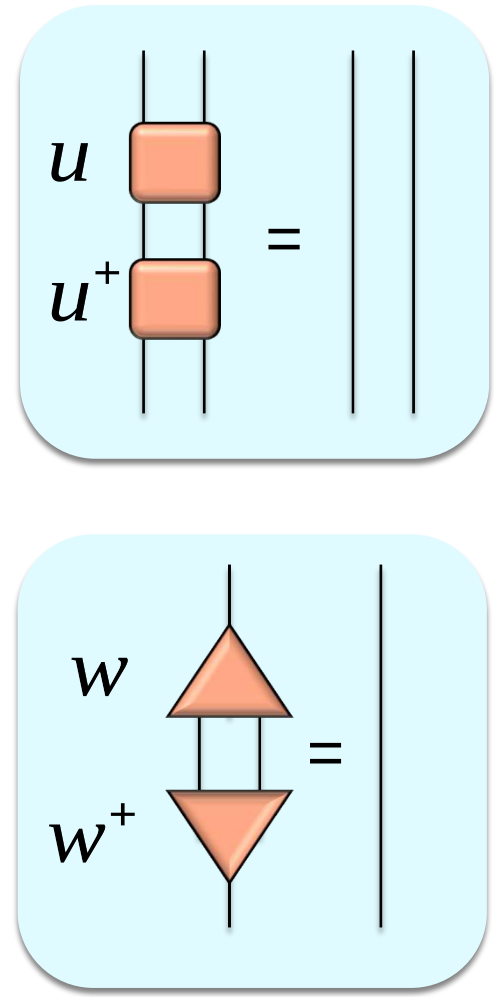


$|\Psi\rangle$

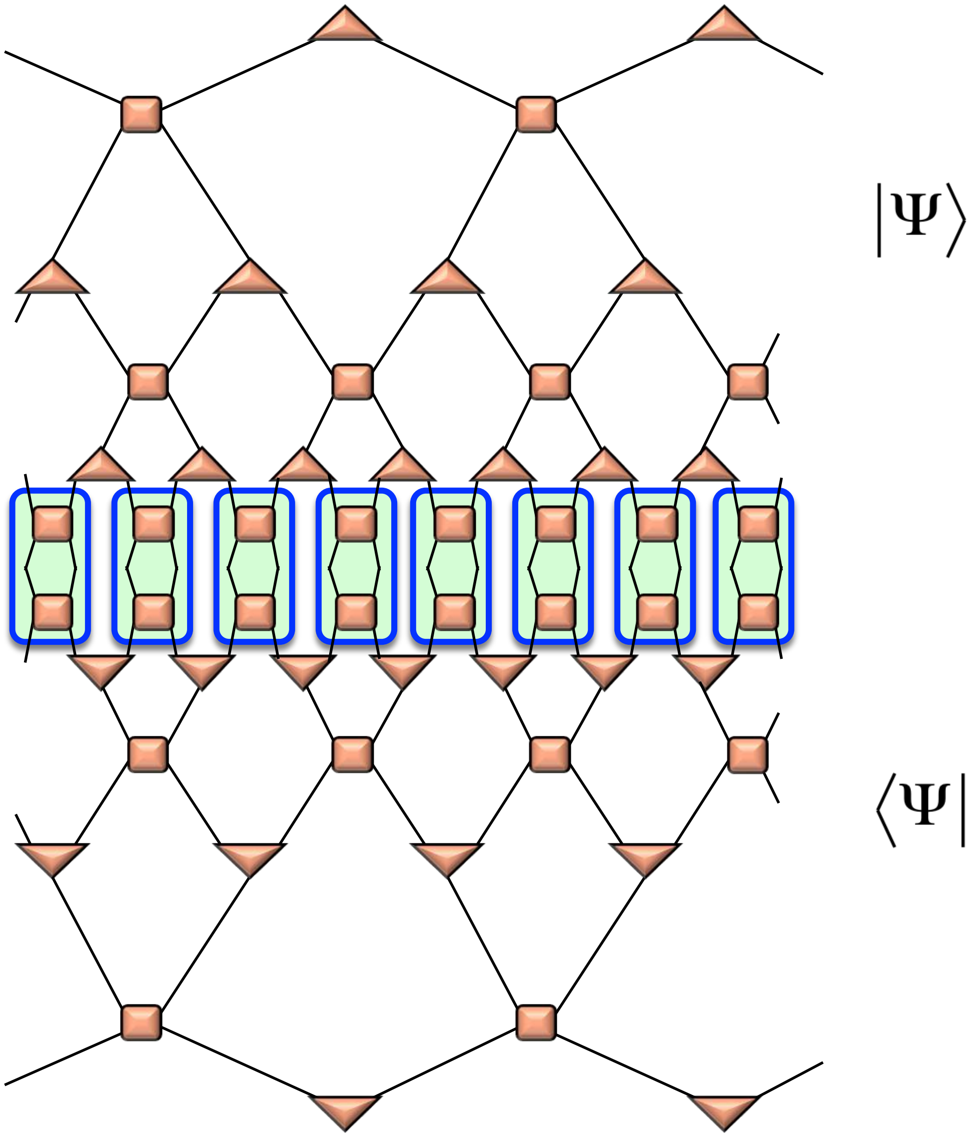
$\langle\Psi|$

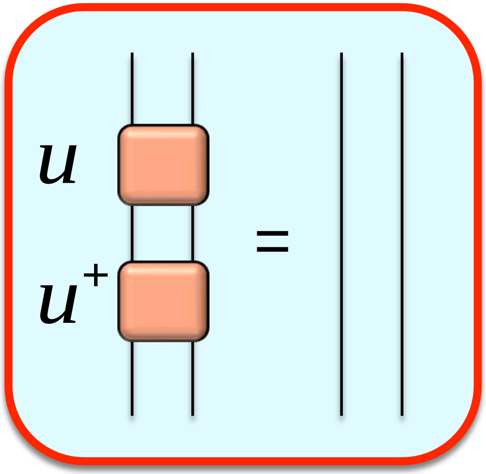
Norm of MERA

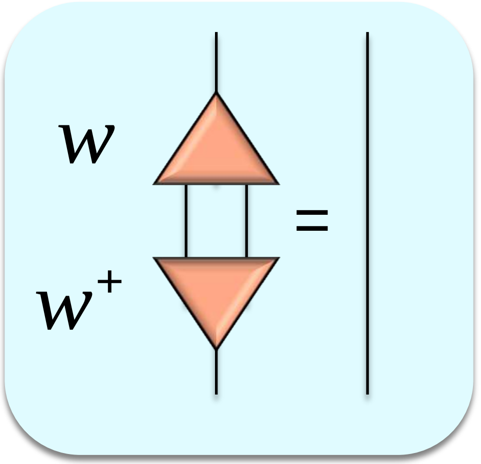




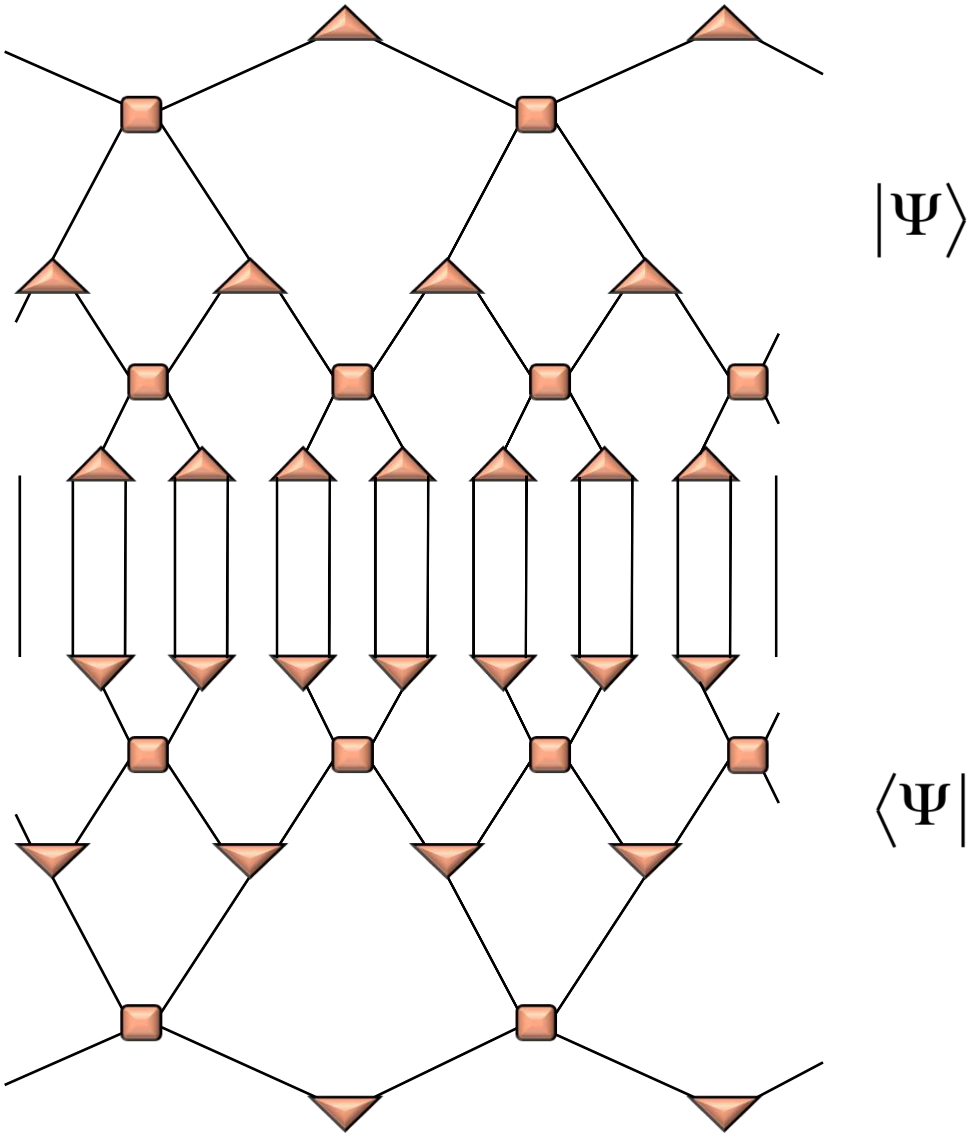
Norm of MERA

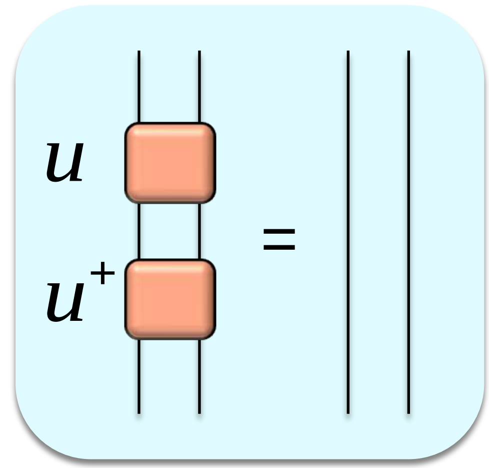


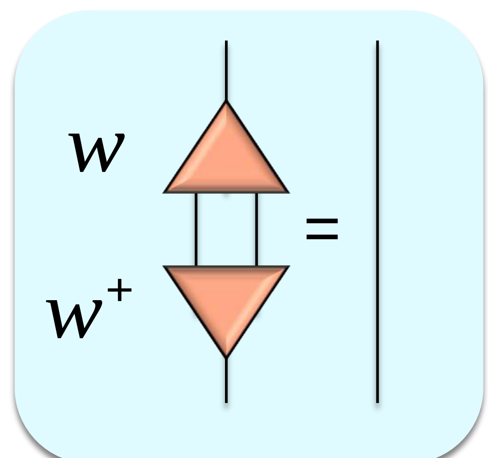




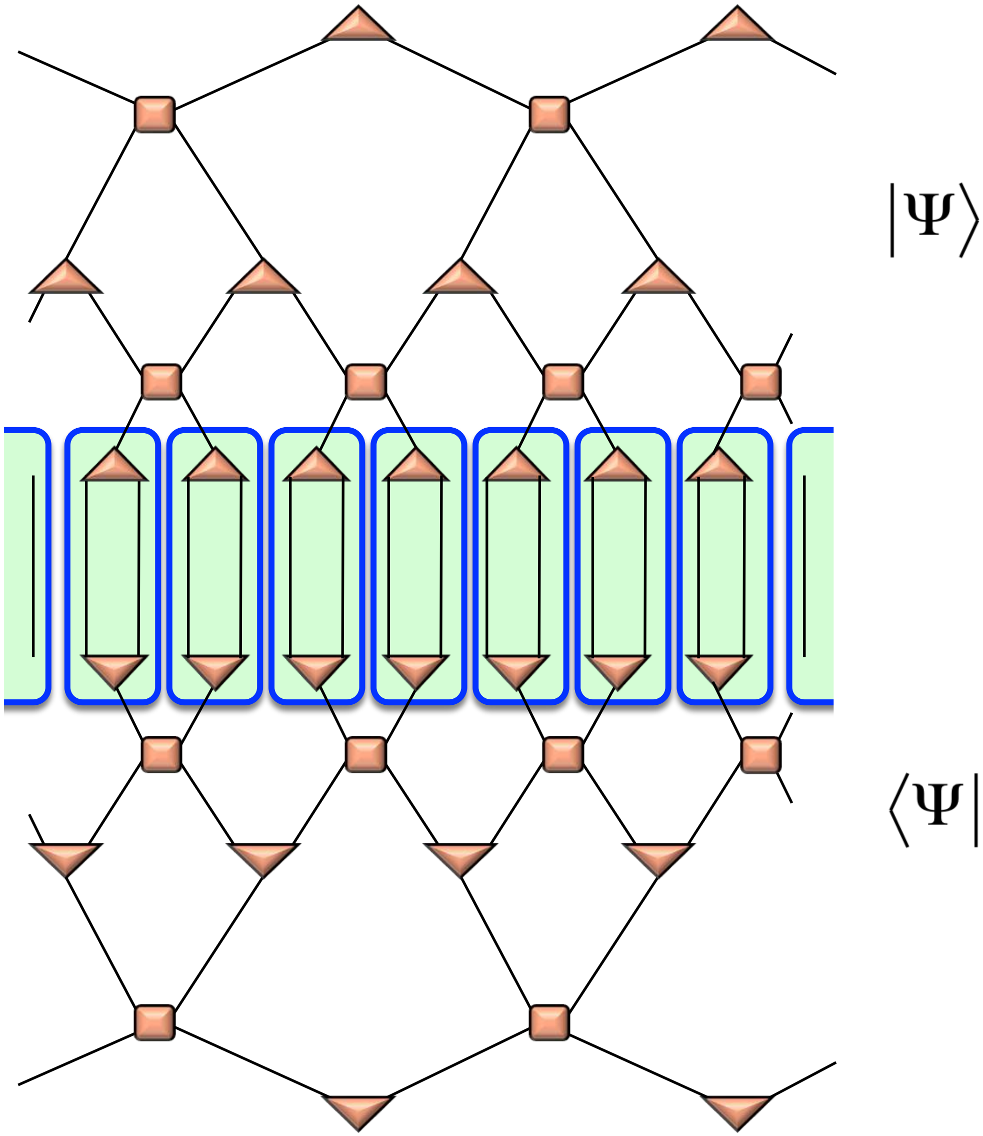
Norm of MERA

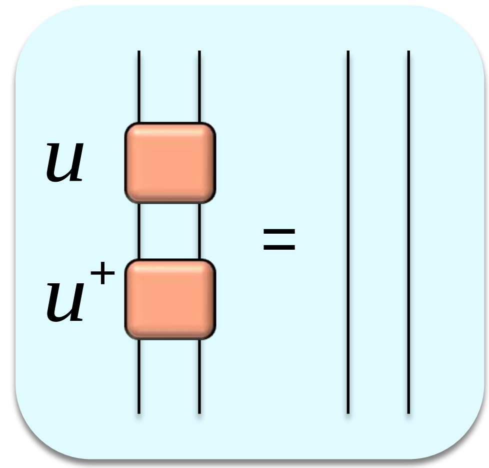


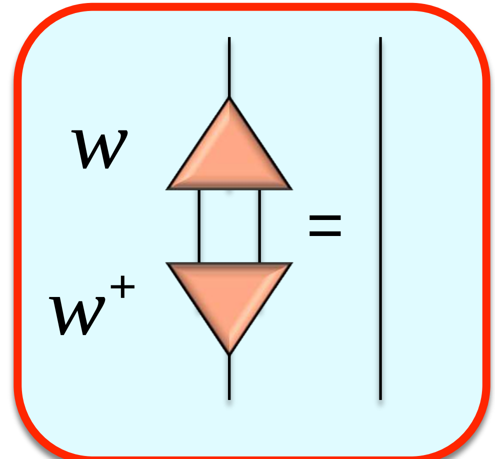




Norm of MERA







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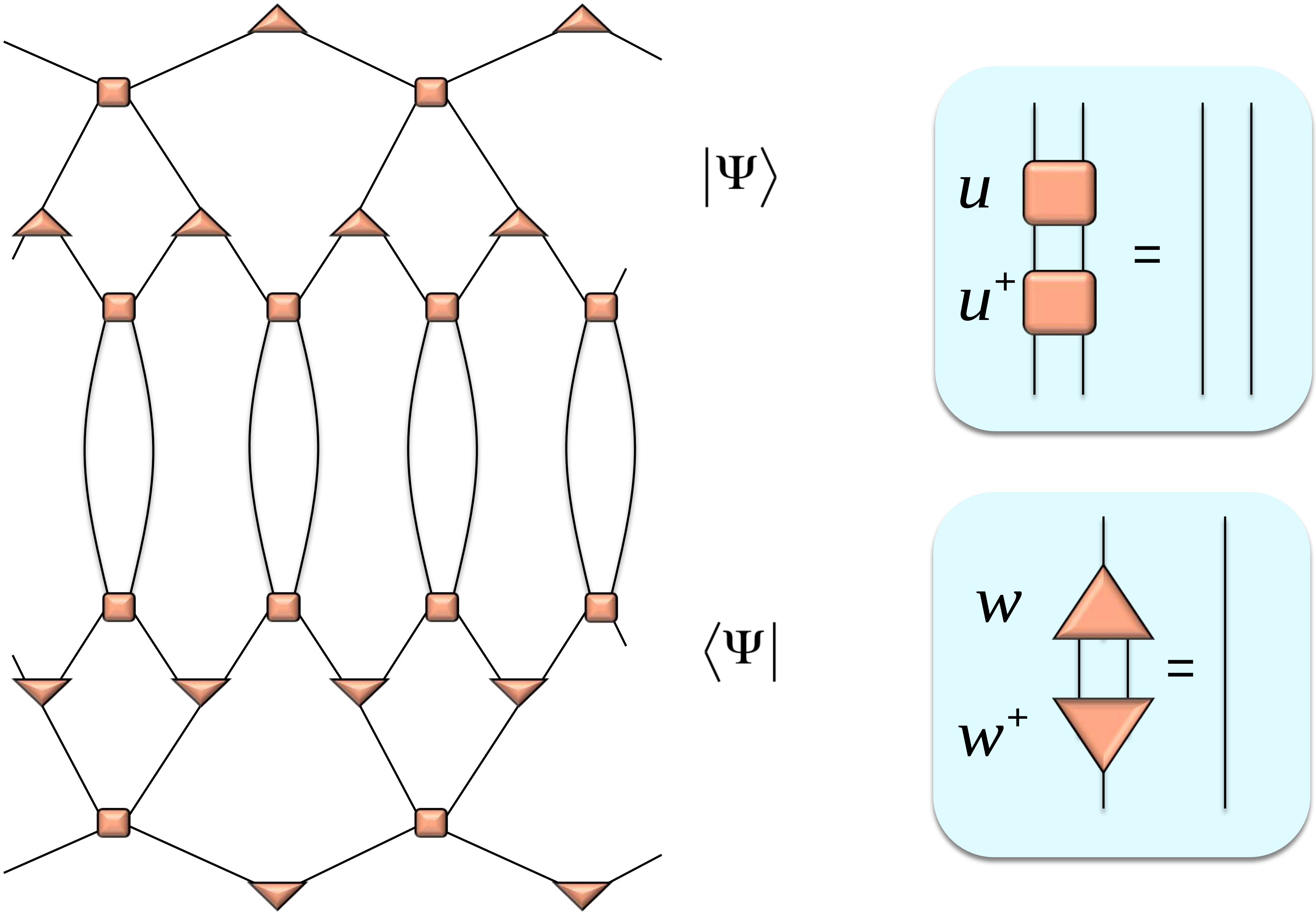
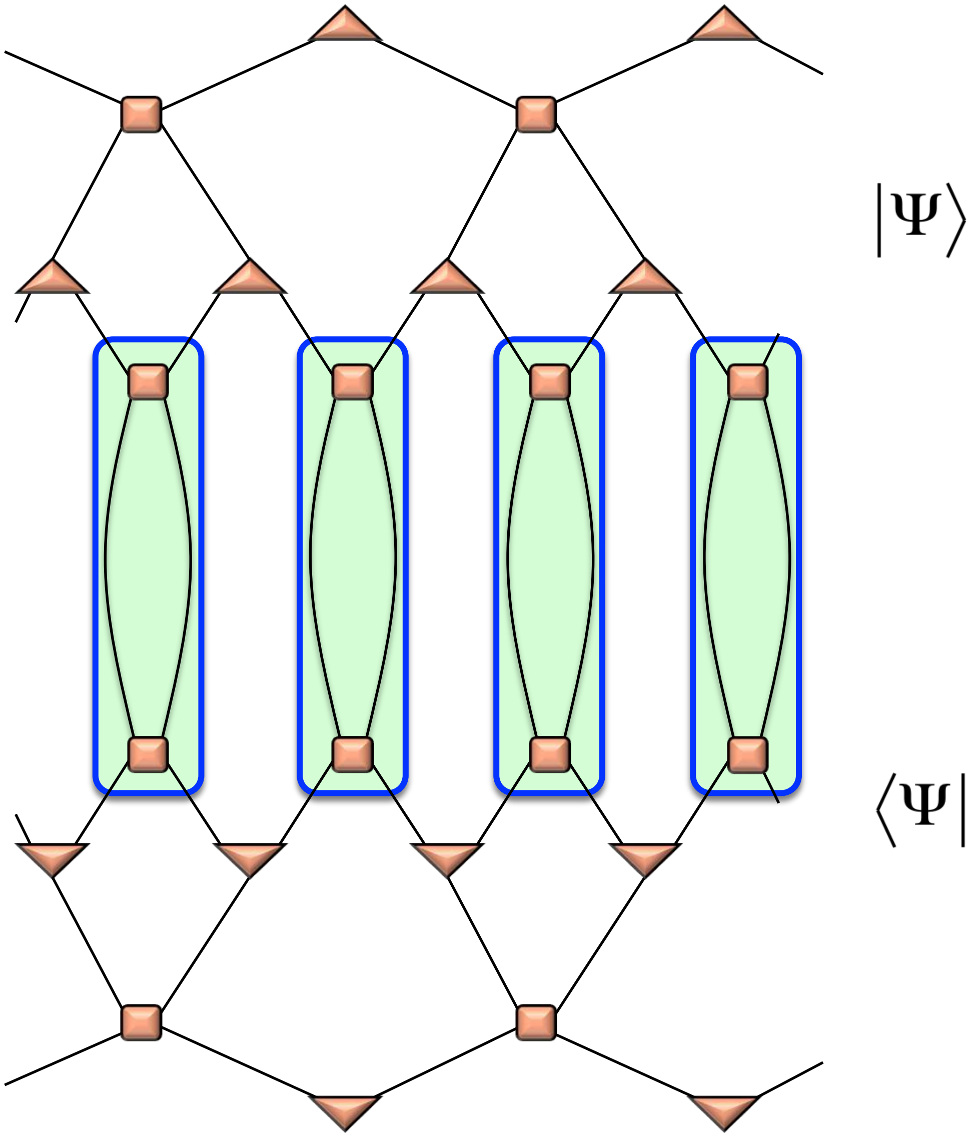
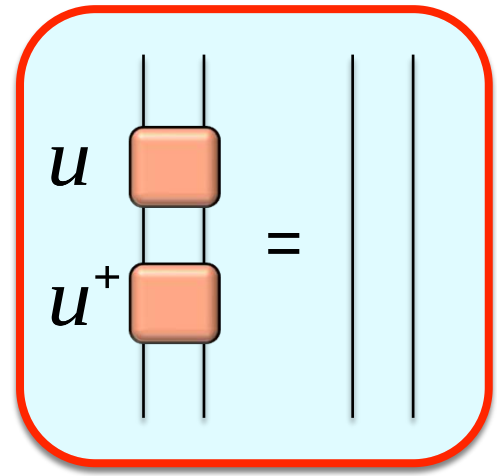
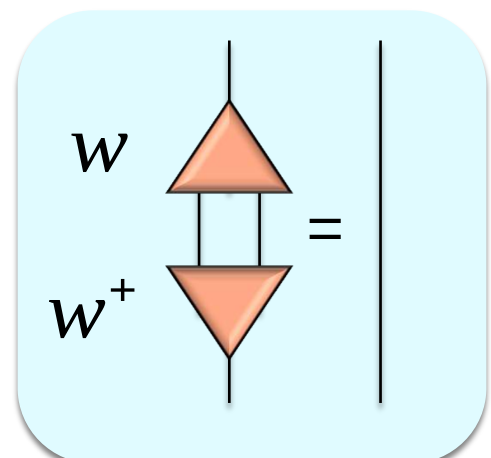


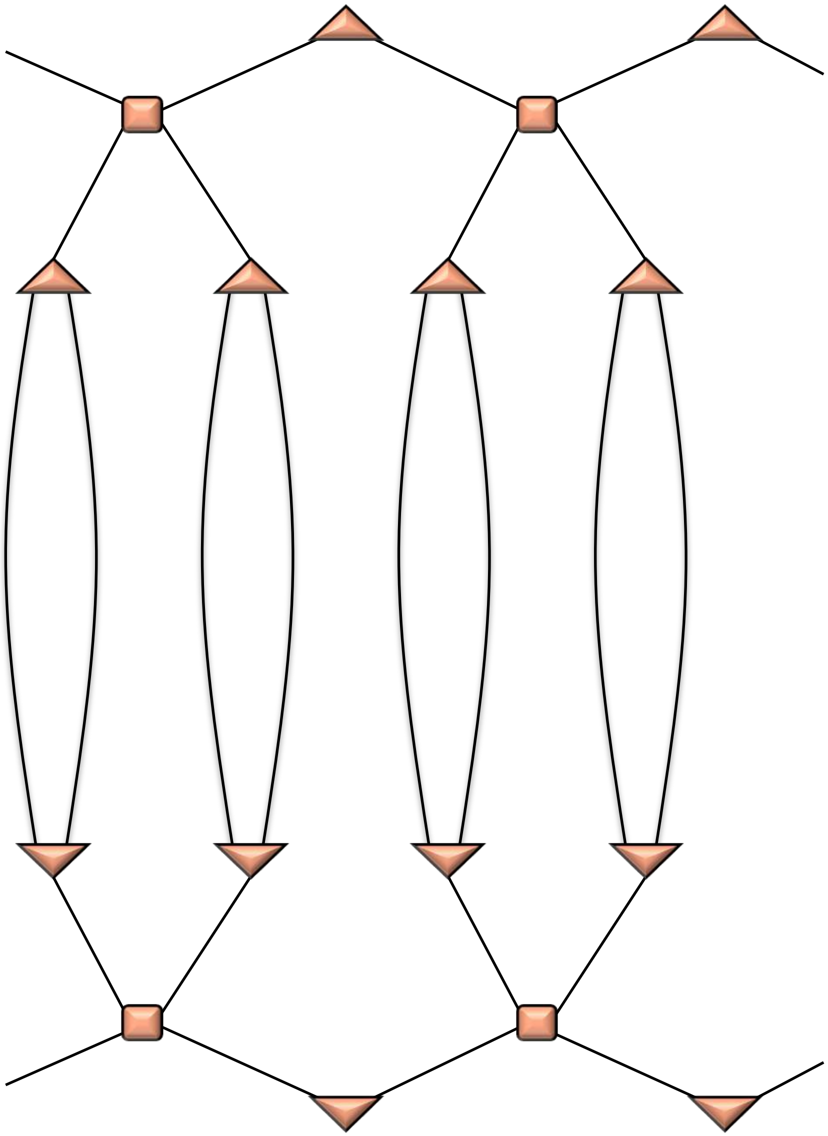
Norm of MERA





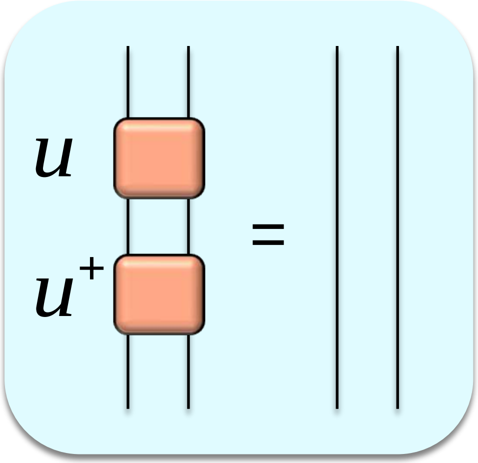


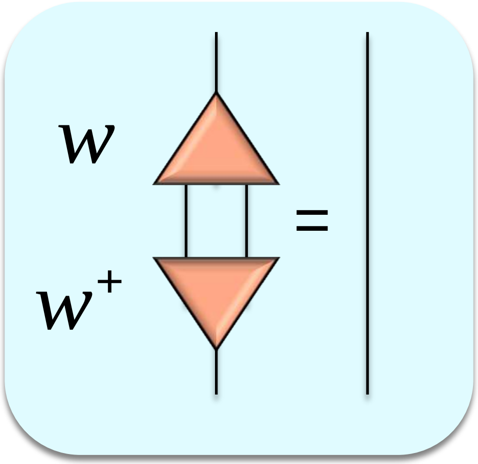
Norm of MERA



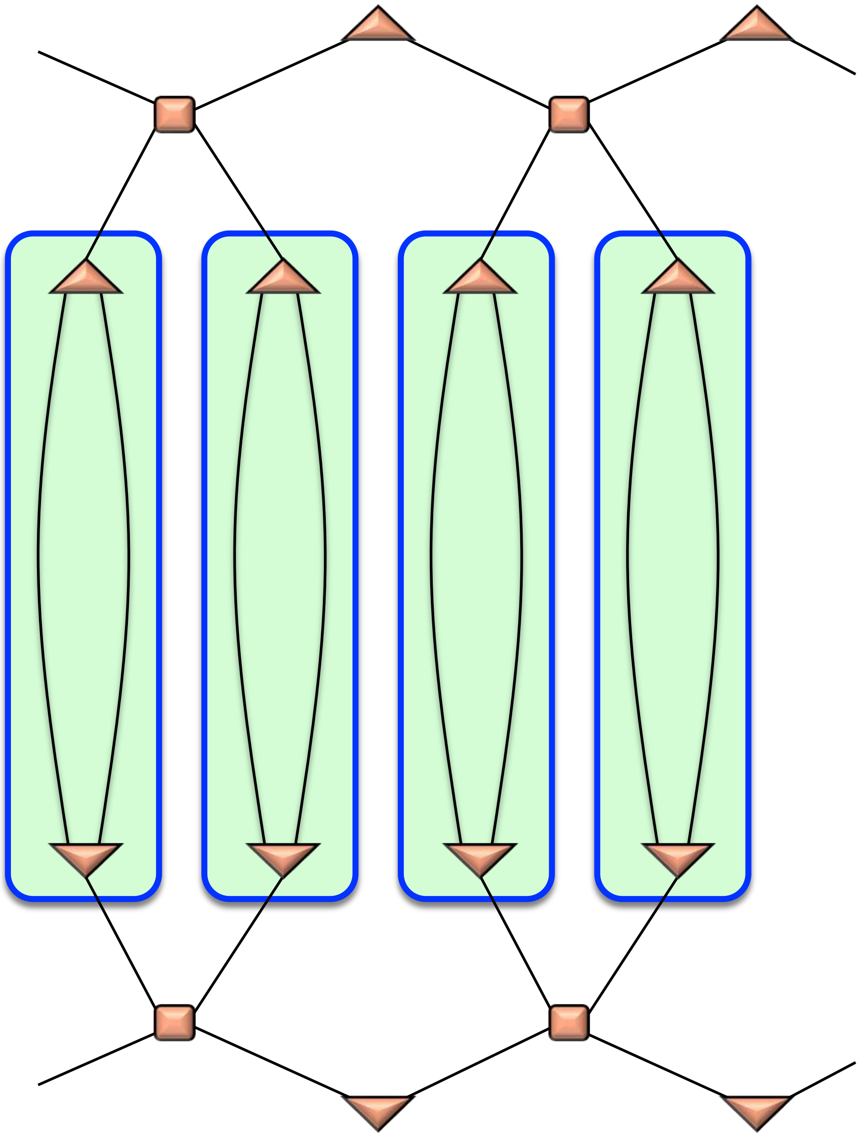
$|\Psi\rangle$

$\langle\Psi|$



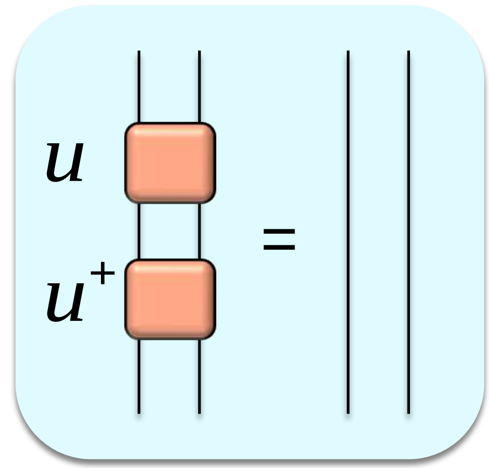


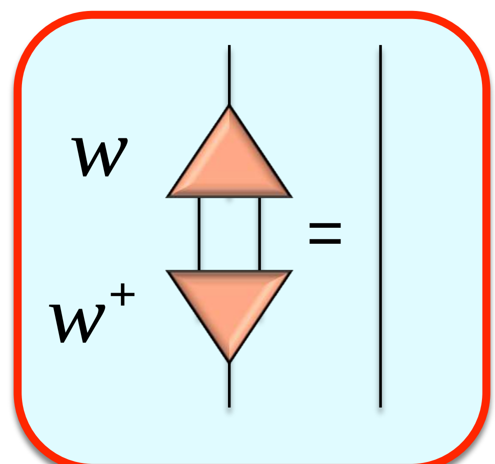
Norm of MERA



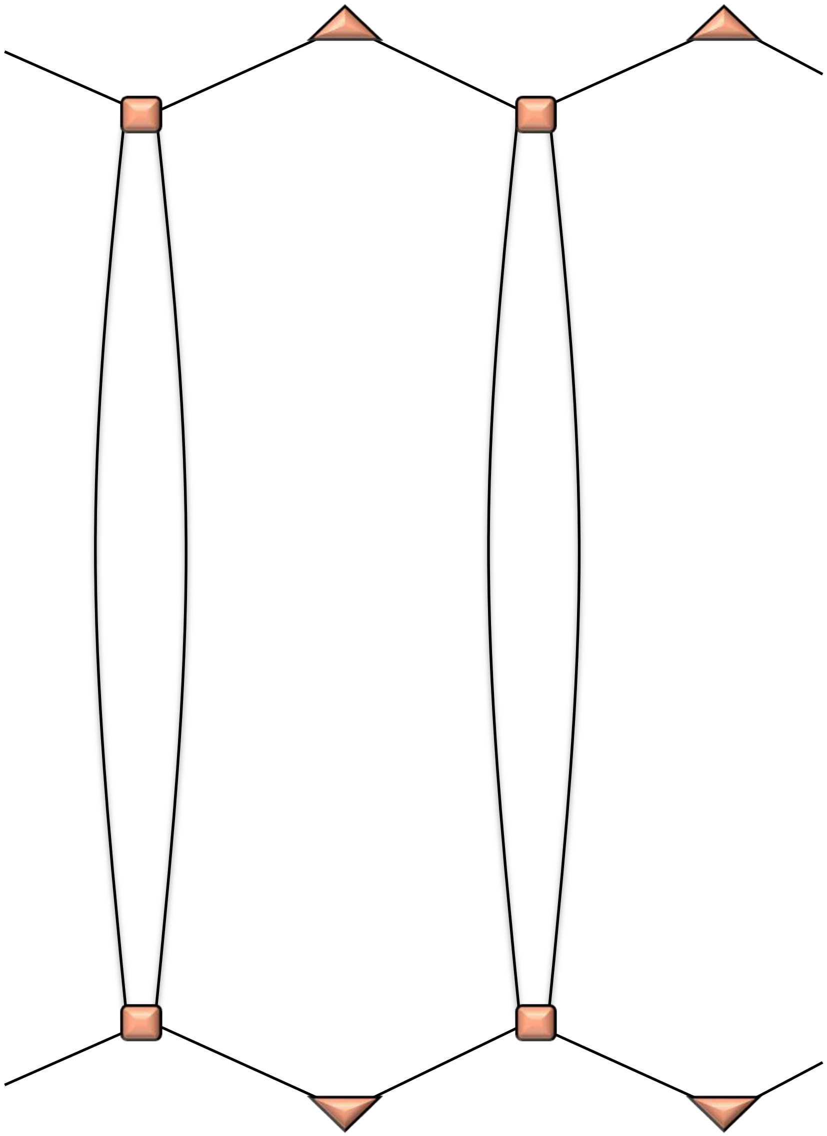
$|\Psi\rangle$

$\langle\Psi|$



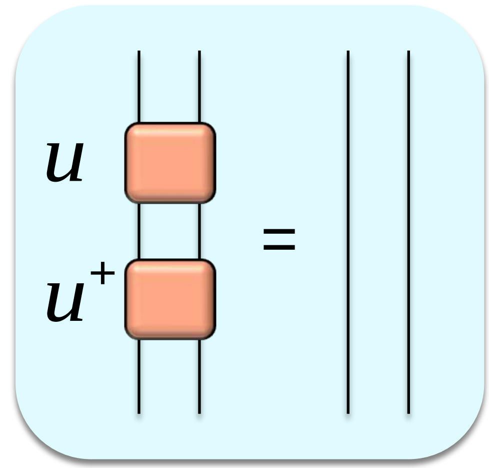


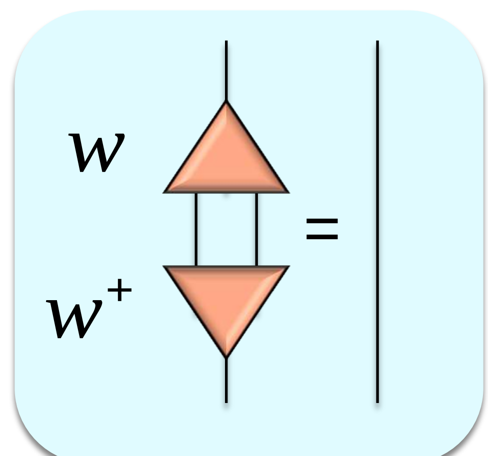
Norm of MERA



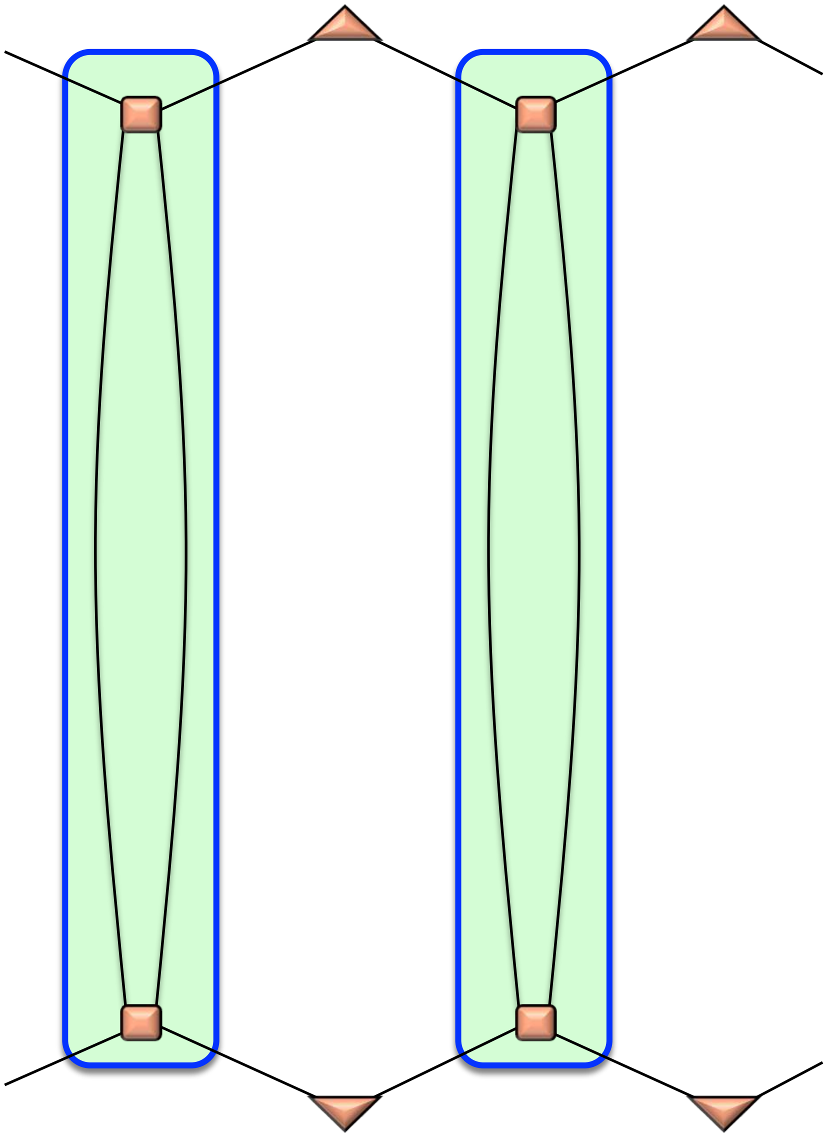
$|\Psi\rangle$

$\langle\Psi|$



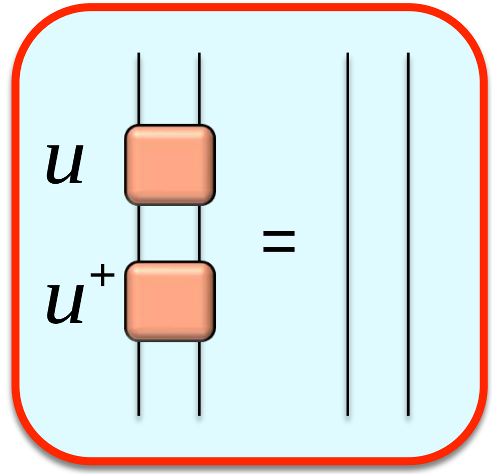


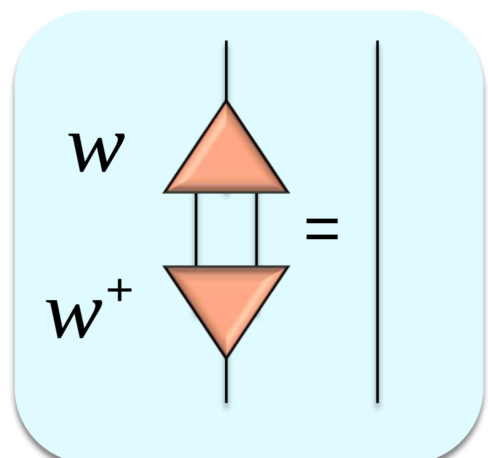
Norm of MERA



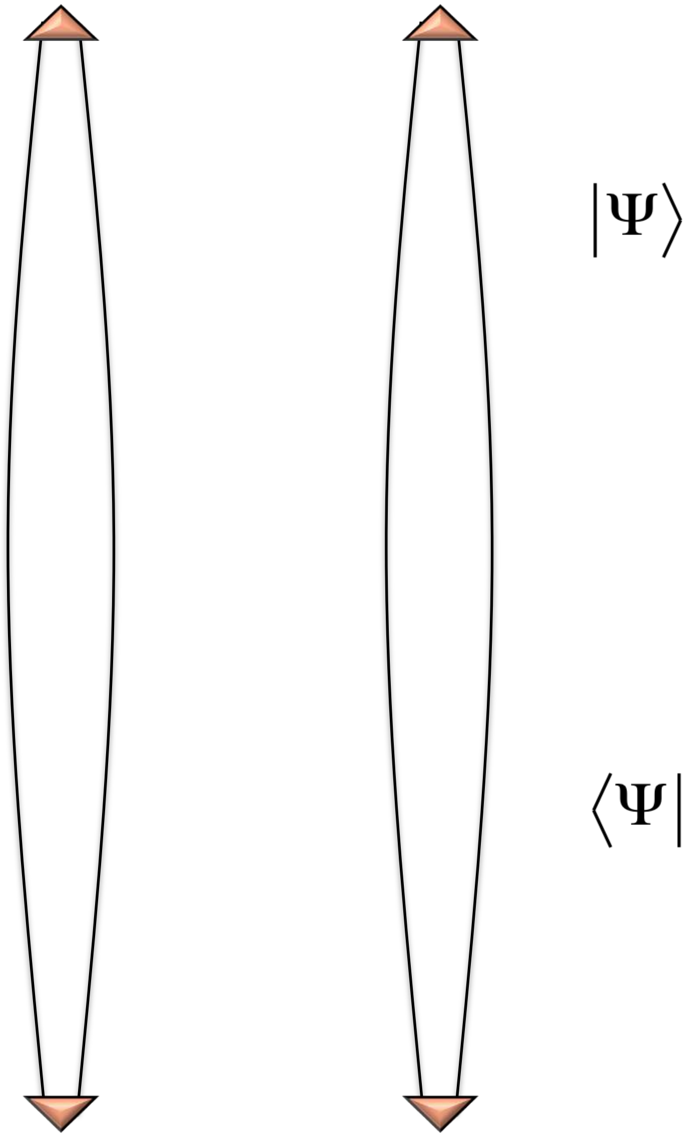
$|\Psi\rangle$

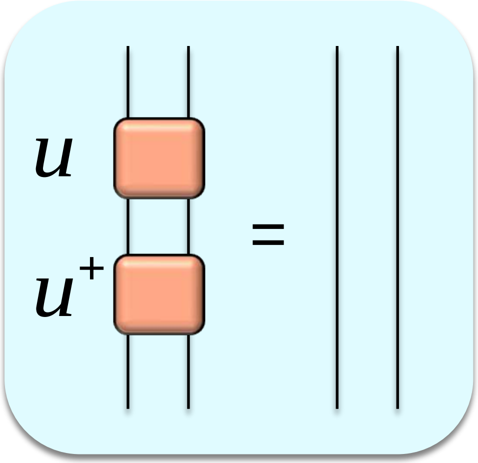
$\langle\Psi|$

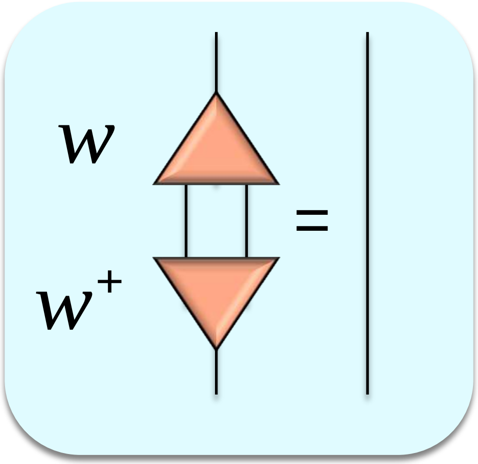




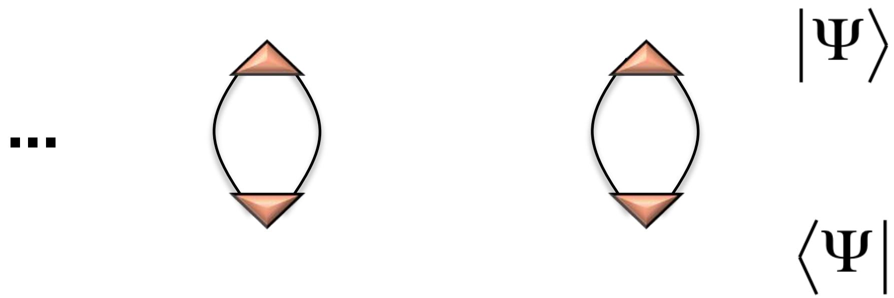
Norm of MERA

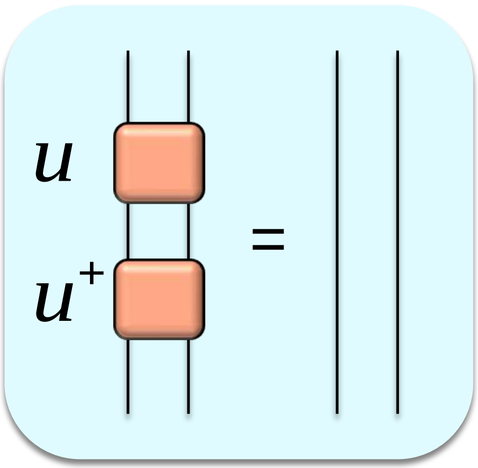


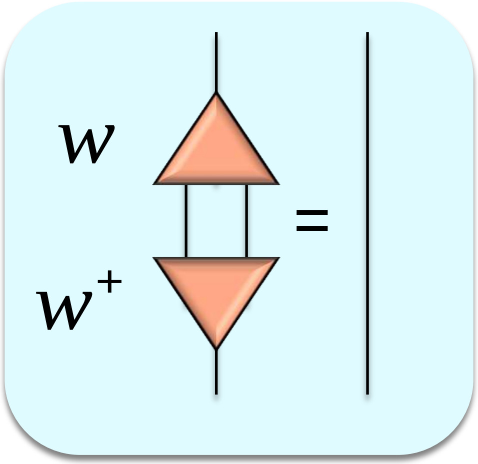




Norm of MERA

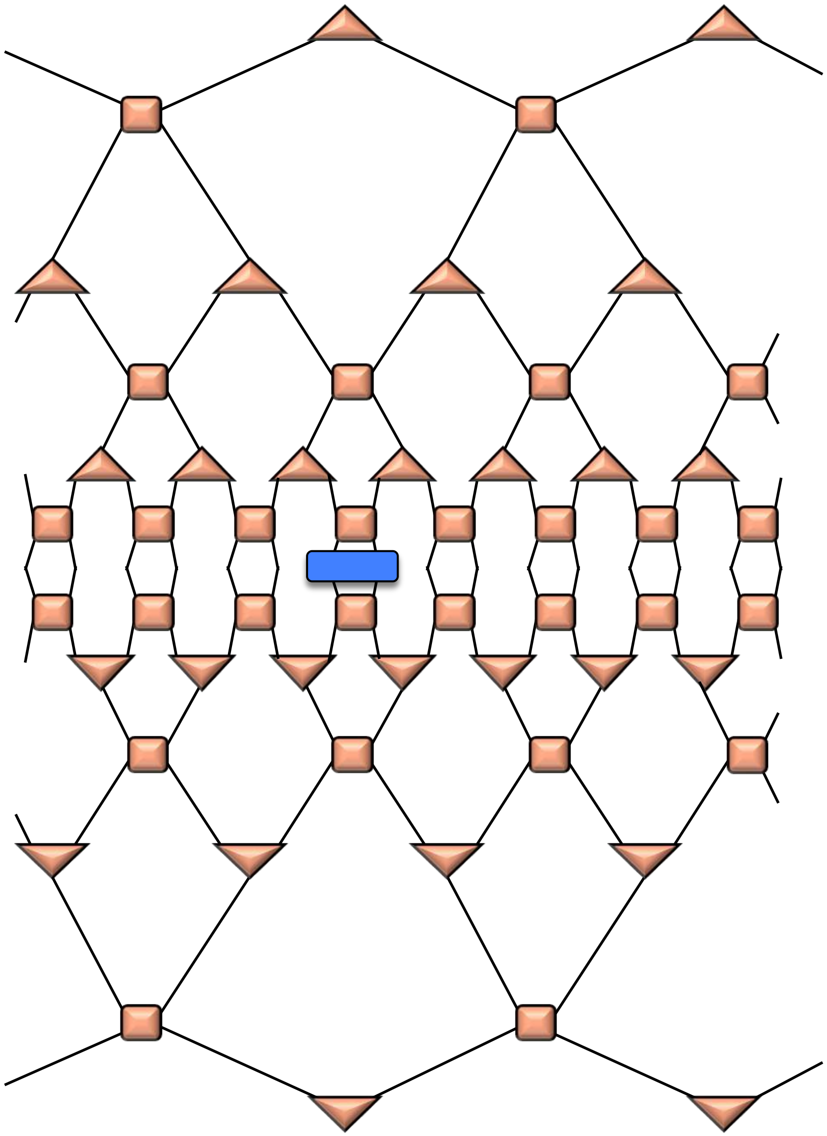






The norm is just the contraction of the top tensors

Expectation values

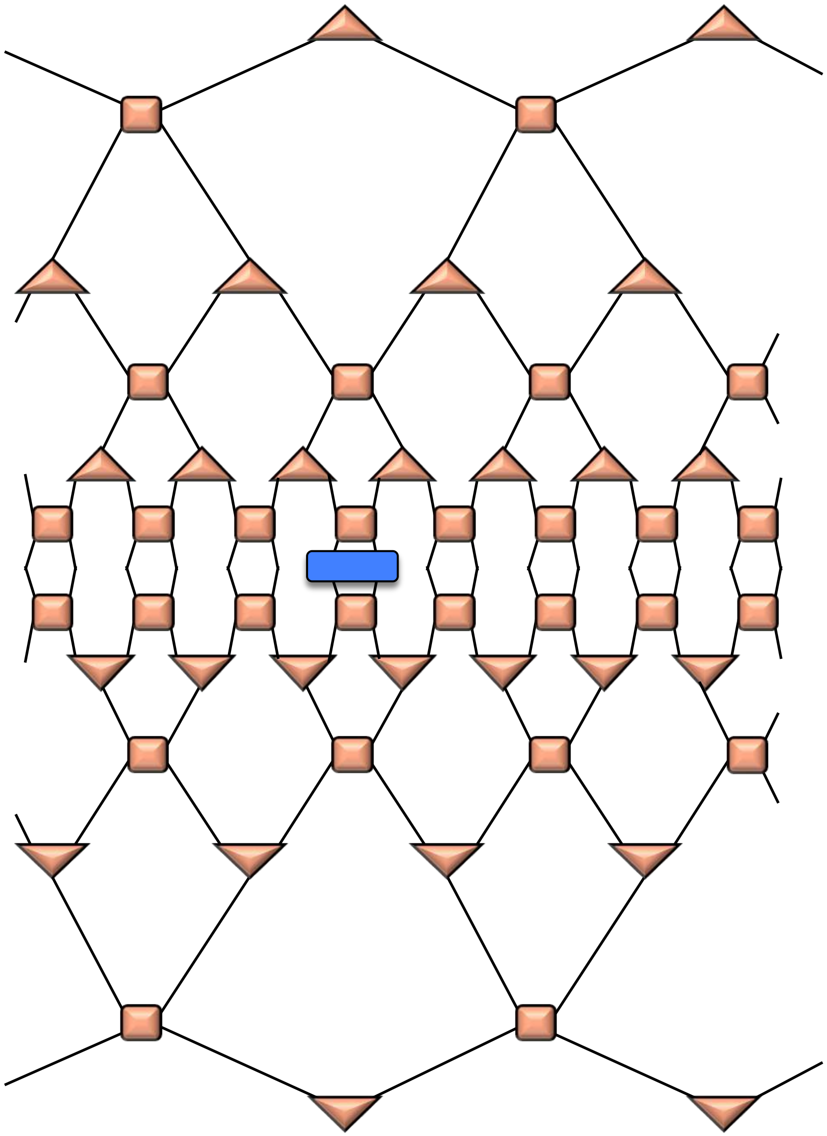


$|\Psi\rangle$

o_{12}

$\langle\Psi|$

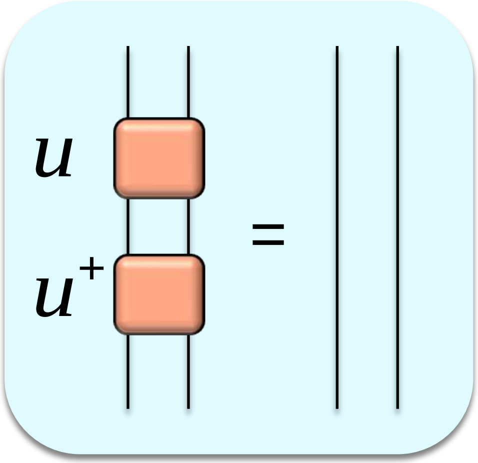
Expectation values

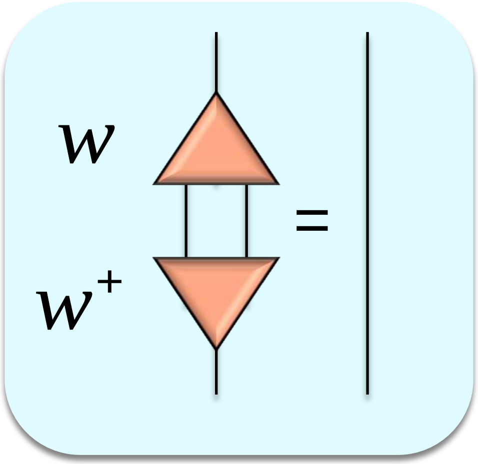


$|\Psi\rangle$

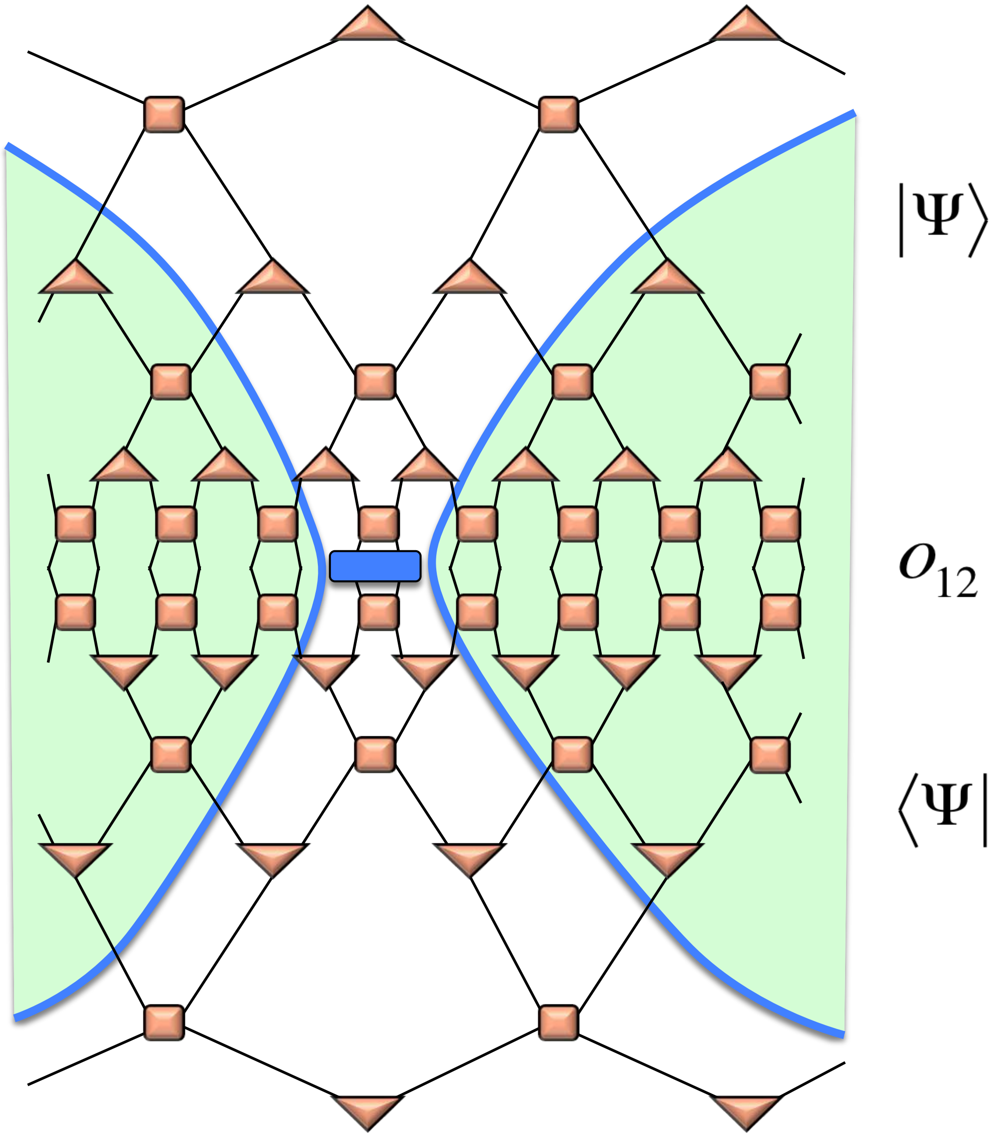
o_{12}

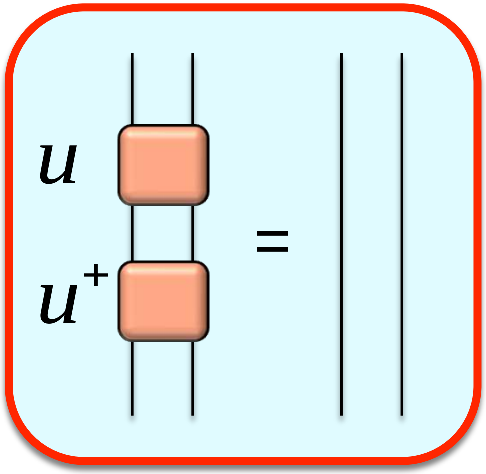
$\langle\Psi|$

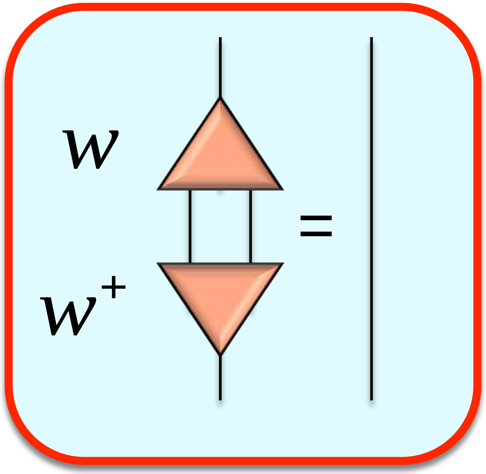




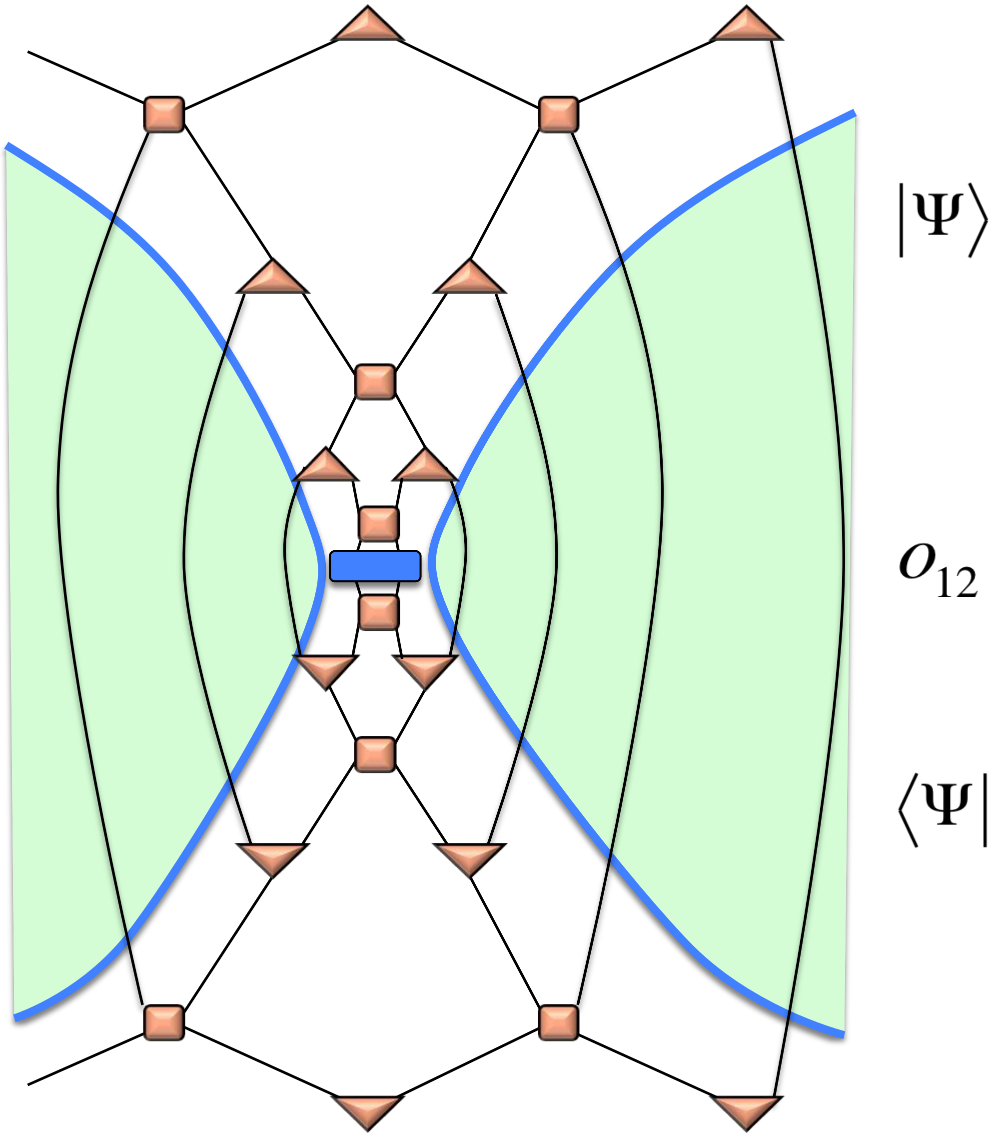
Expectation values

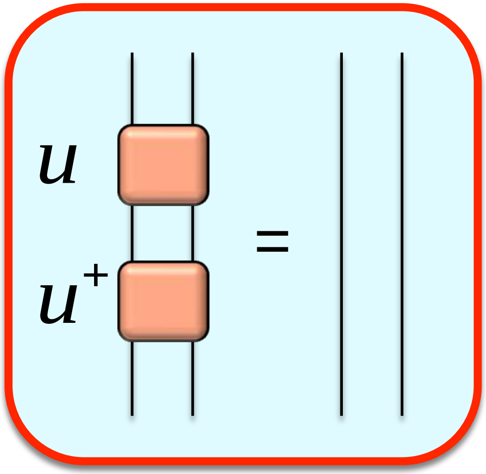


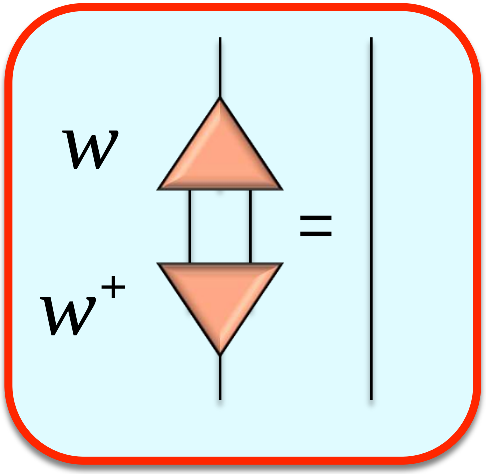




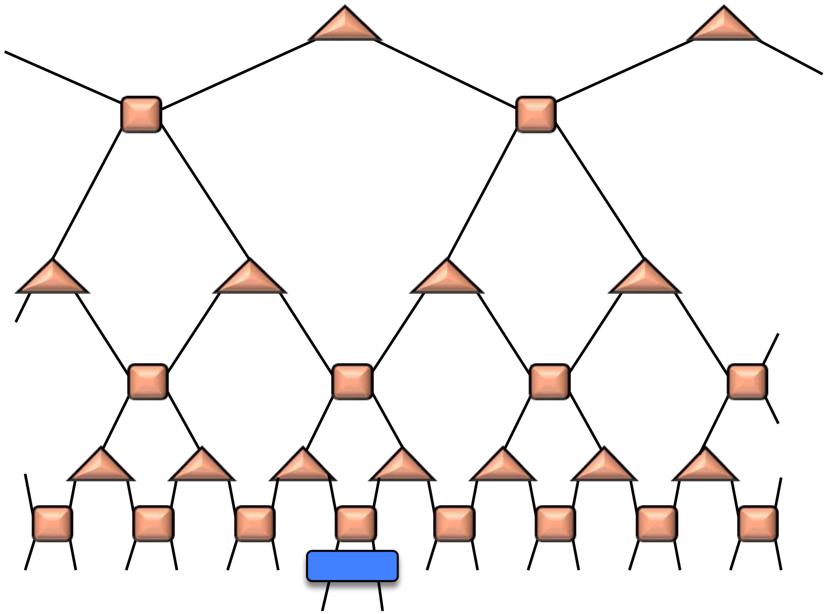
Expectation values







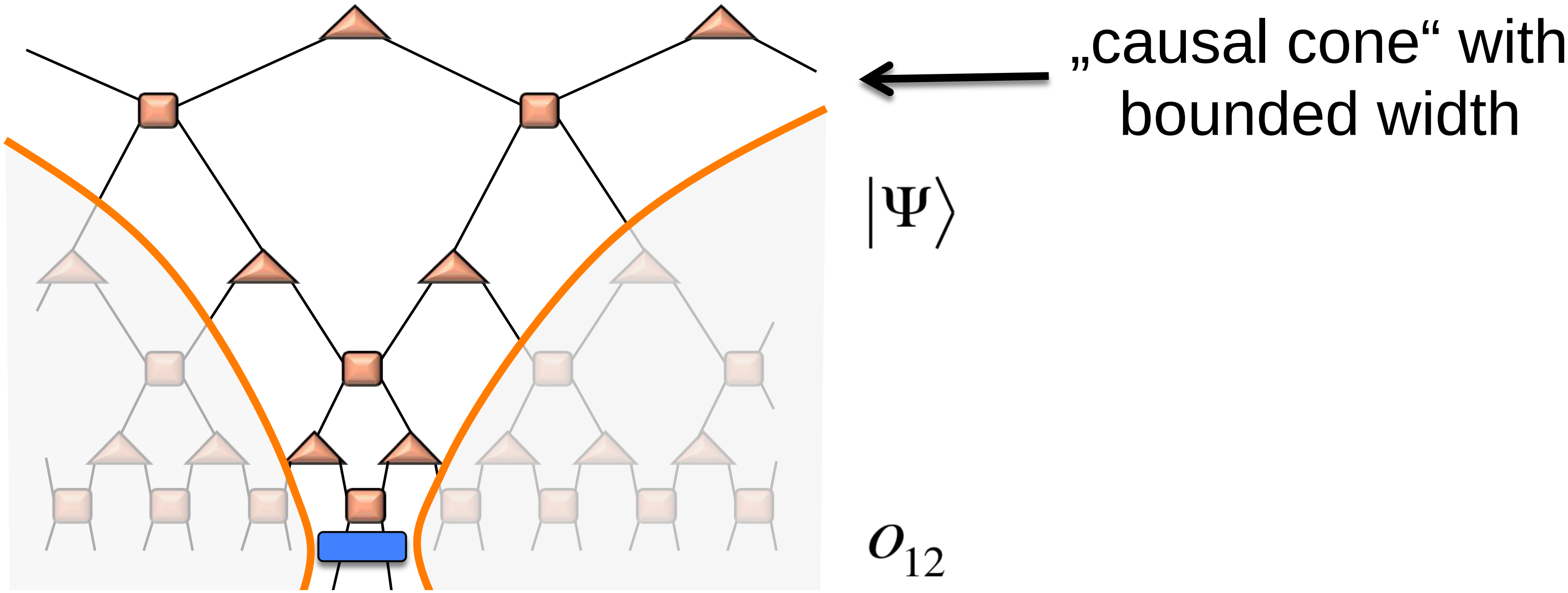
Expectation values



$|\Psi\rangle$

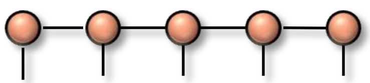
o_{12}

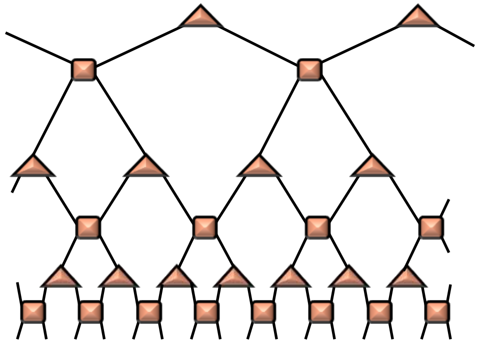
Expectation values



Only tensors inside of the causal cone contribute to the expectation value

MPS vs 1d MERA





MPS in 1d

1d area law $S(L) = O(1)$

Exact contraction is efficient

Finite correlation length

to/from 1d Hamiltonians

Arbitrary tensors

MERA in 1d

Beyond 1d area law $S(L) = O(\log L)$

Exact contraction is efficient

Finite and infinite correlation lengths

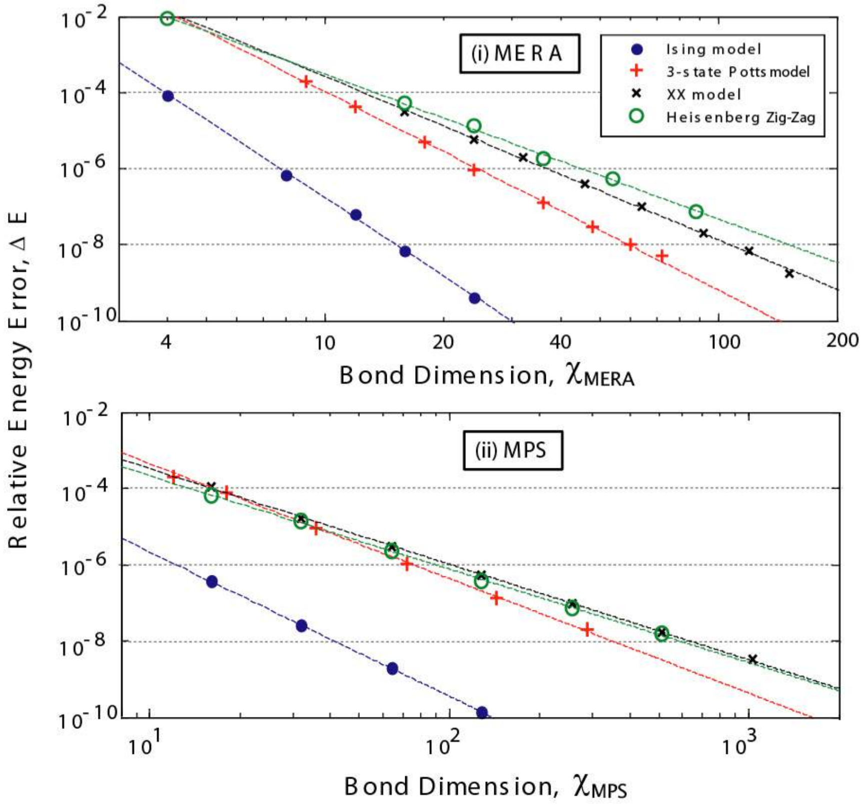
to/from 1d Hamiltonians

Constrained tensors

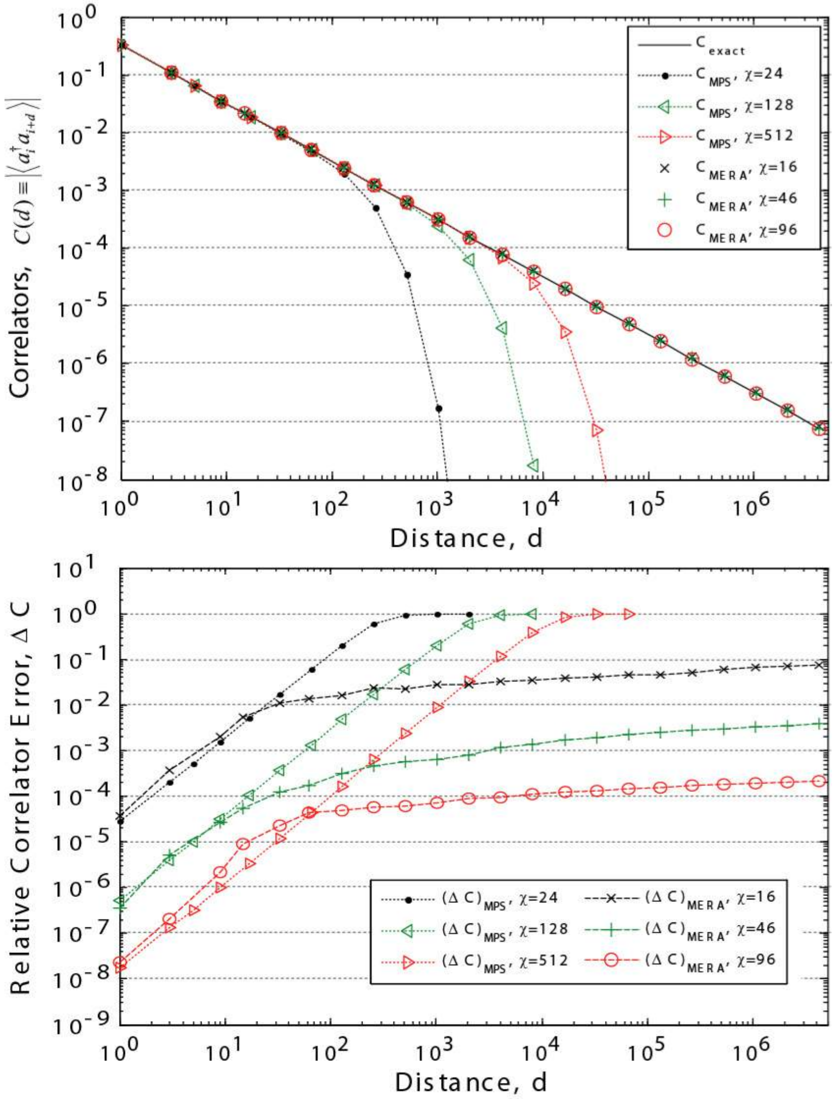
An example:
1d critical systems

G. Evenbly, G. Vidal, in "Strongly Correlated Systems. Numerical Methods", Springer, Vol. 176 (2013)

Critical Energies

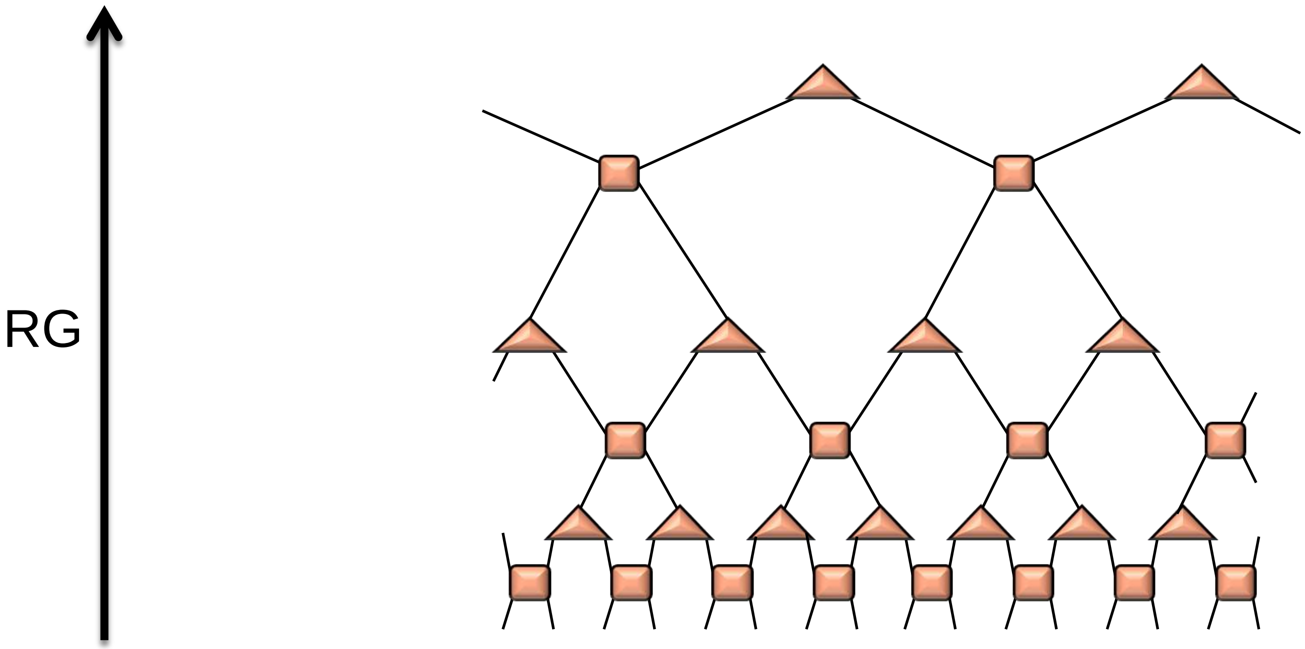


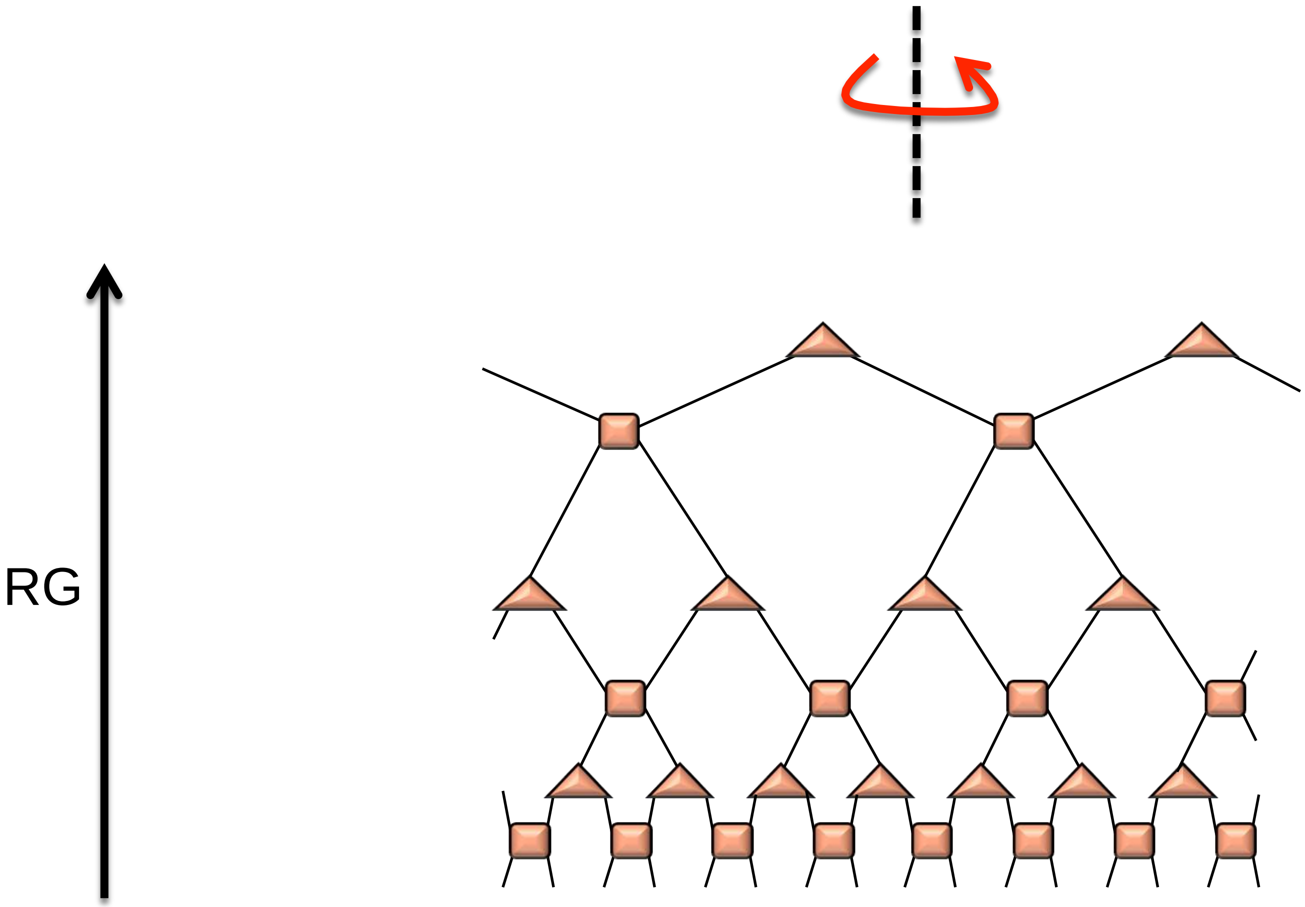
Critical XX Correlators



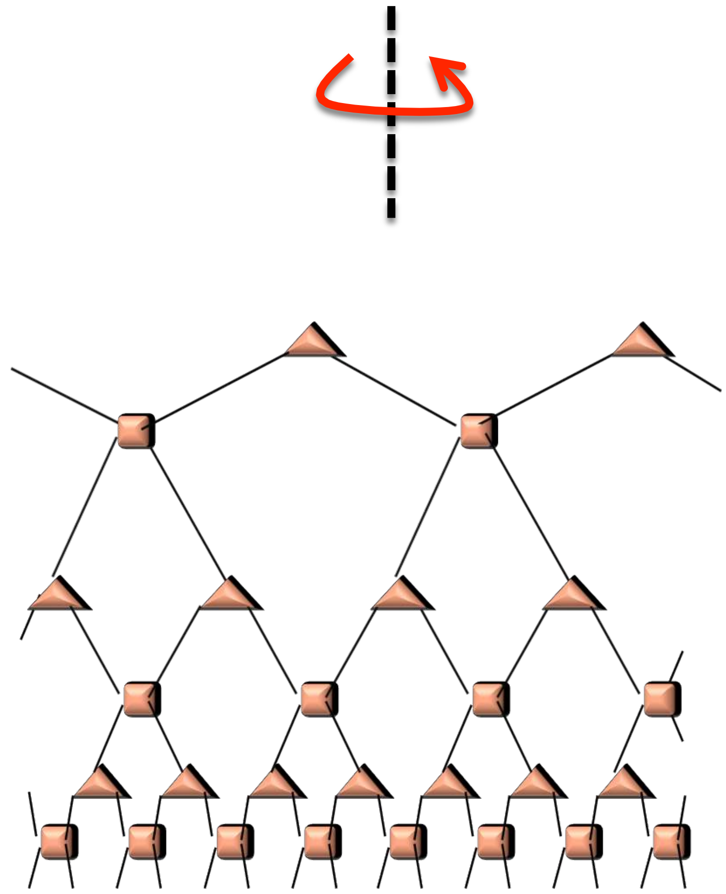
“branching” MERA

G. Evenbly, G. Vidal, arXiv:1210.1895

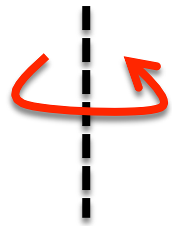

RG

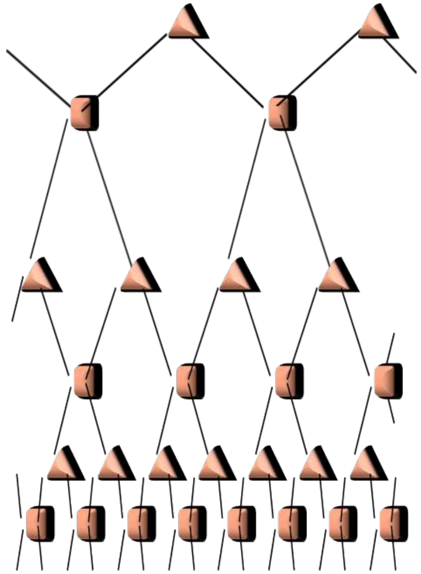

RG

RG

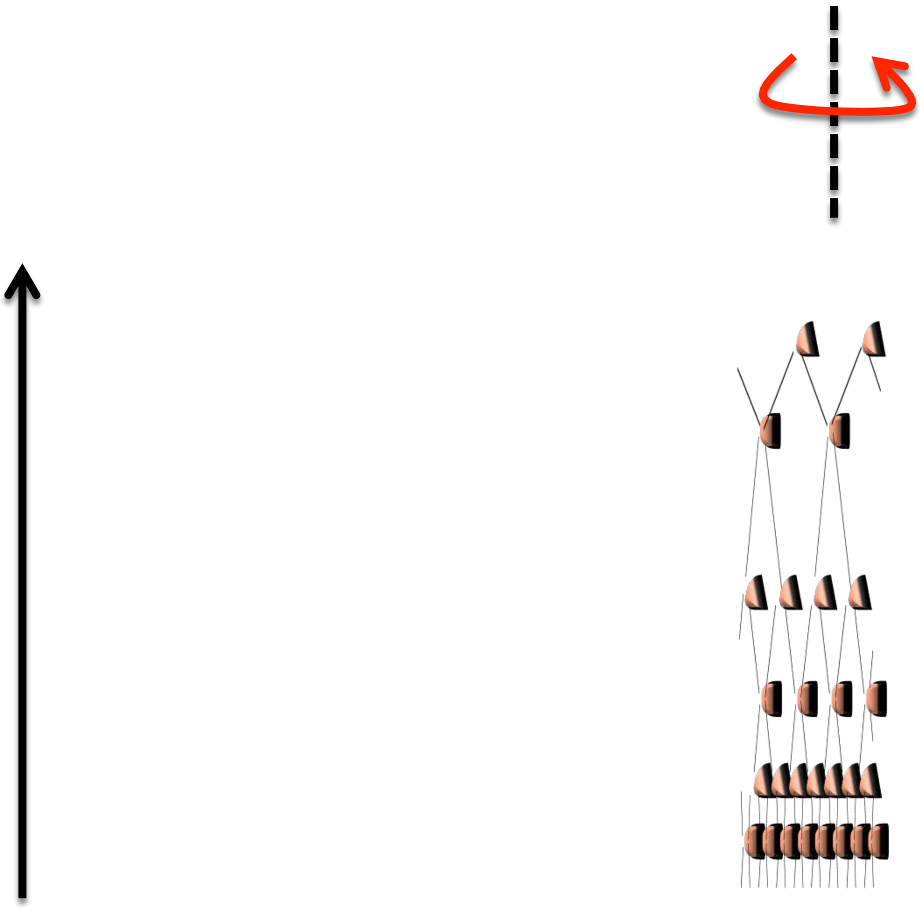


RG



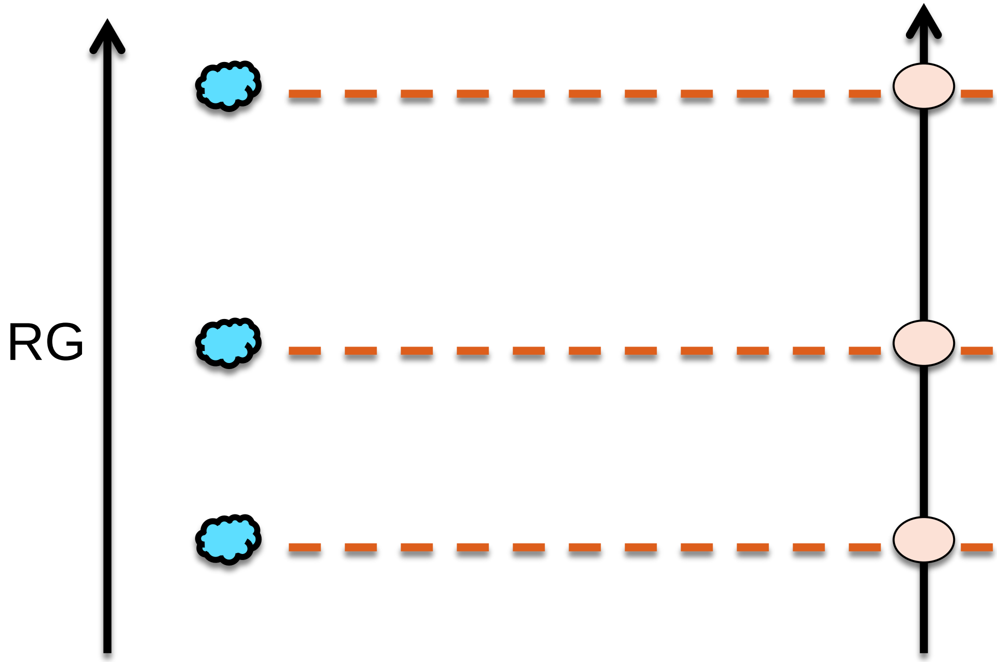


RG

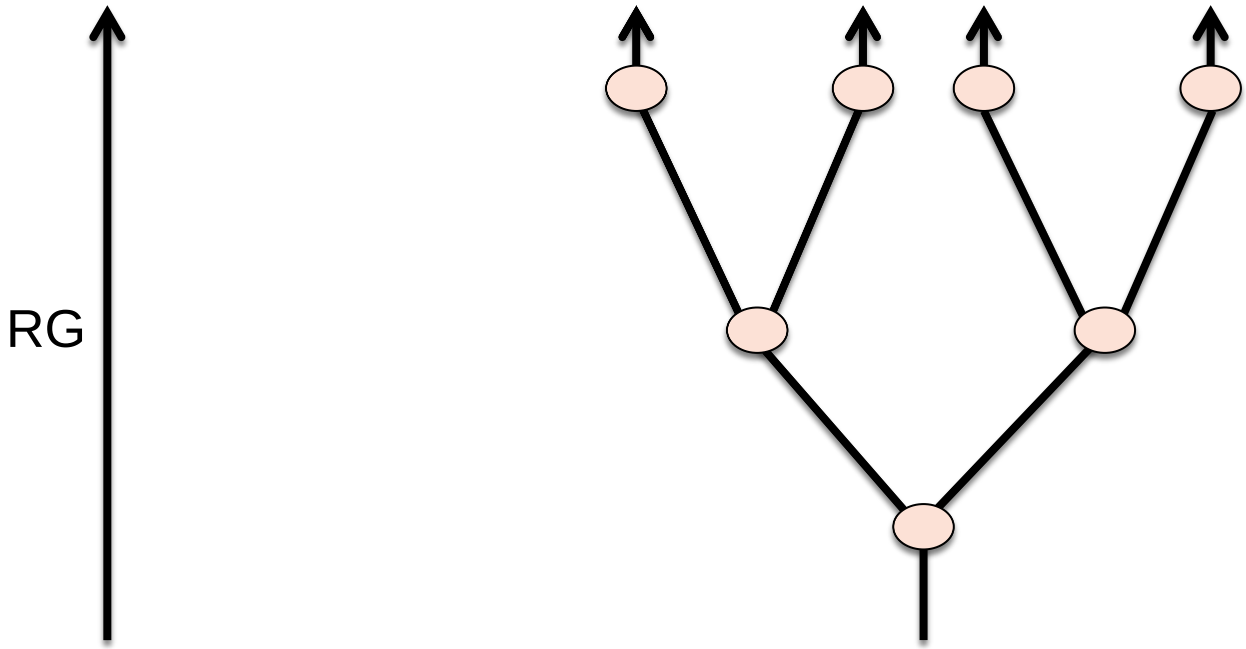


RG

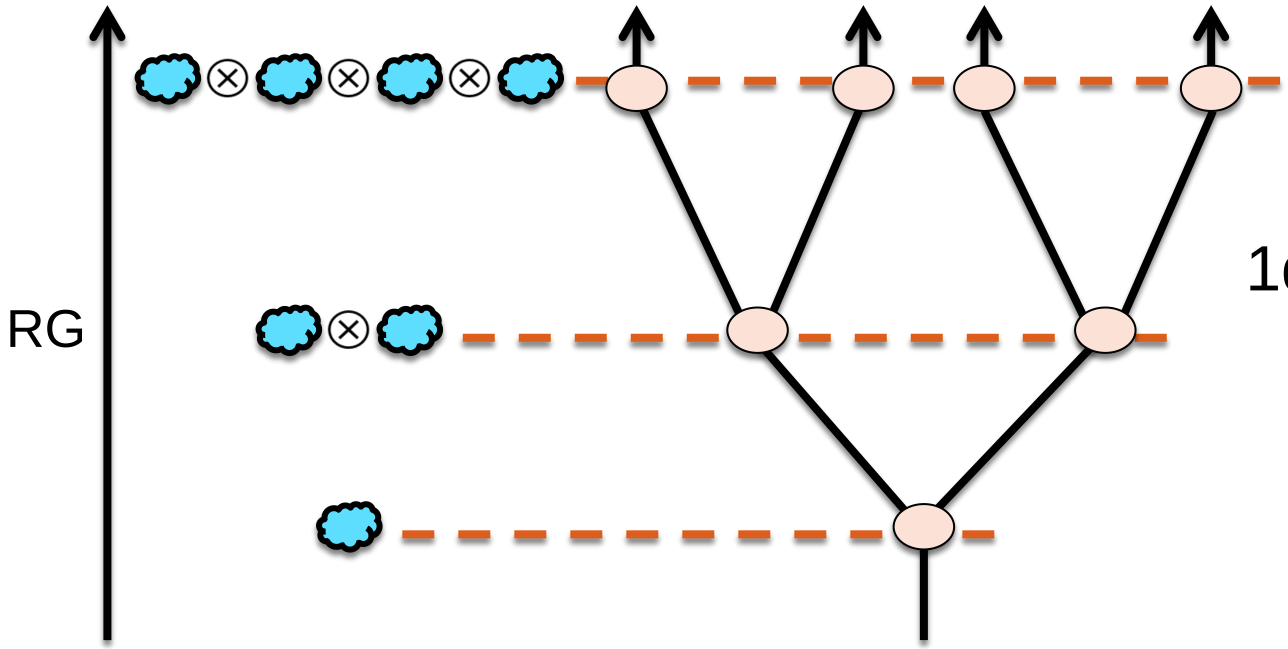
1d MERA



1d MERA



1d branching MERA

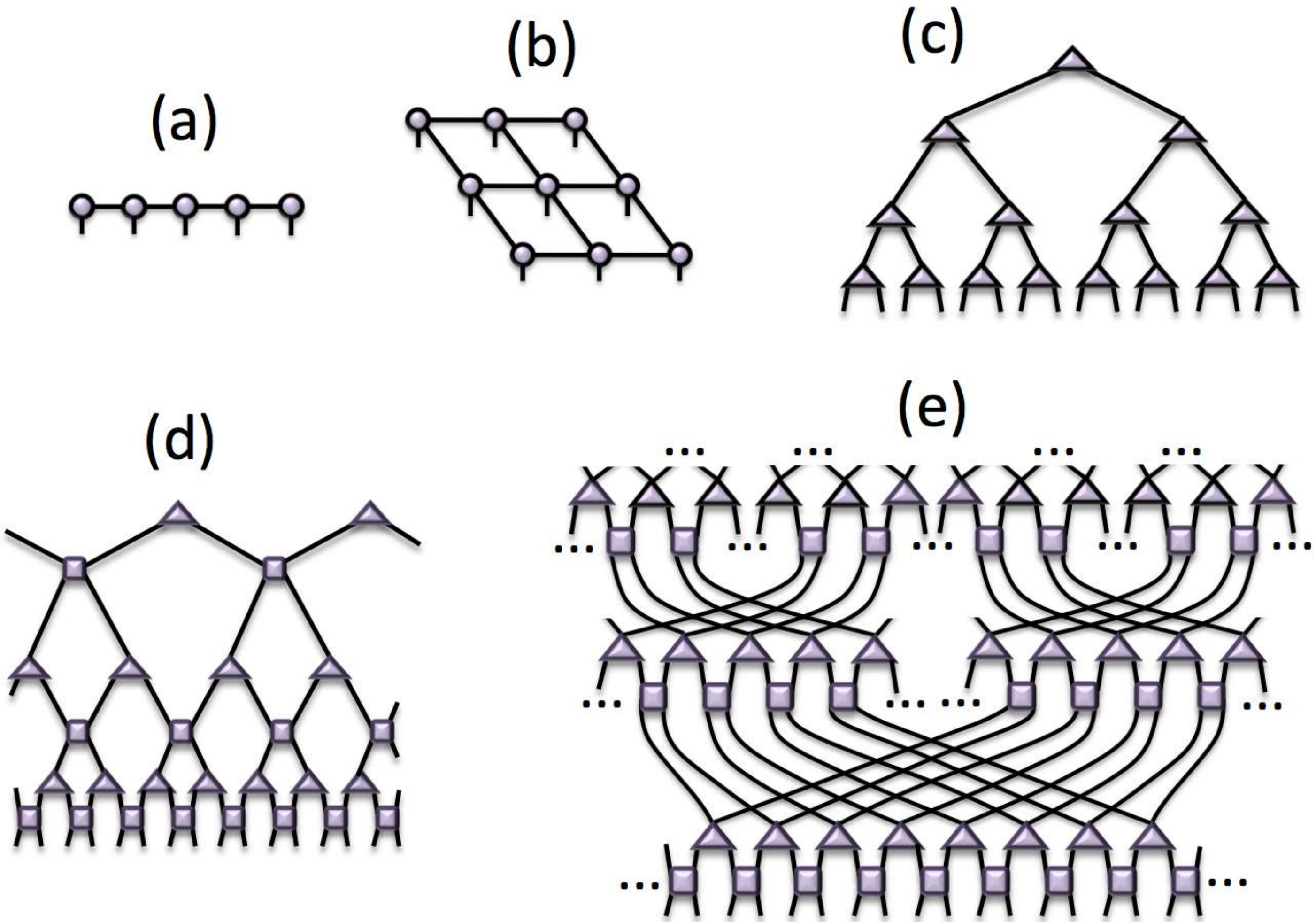


1d branching MERA

Decoupling of degrees of freedom along RG (e.g. spin-charge separation), and allows arbitrary scalings of the entanglement entropy

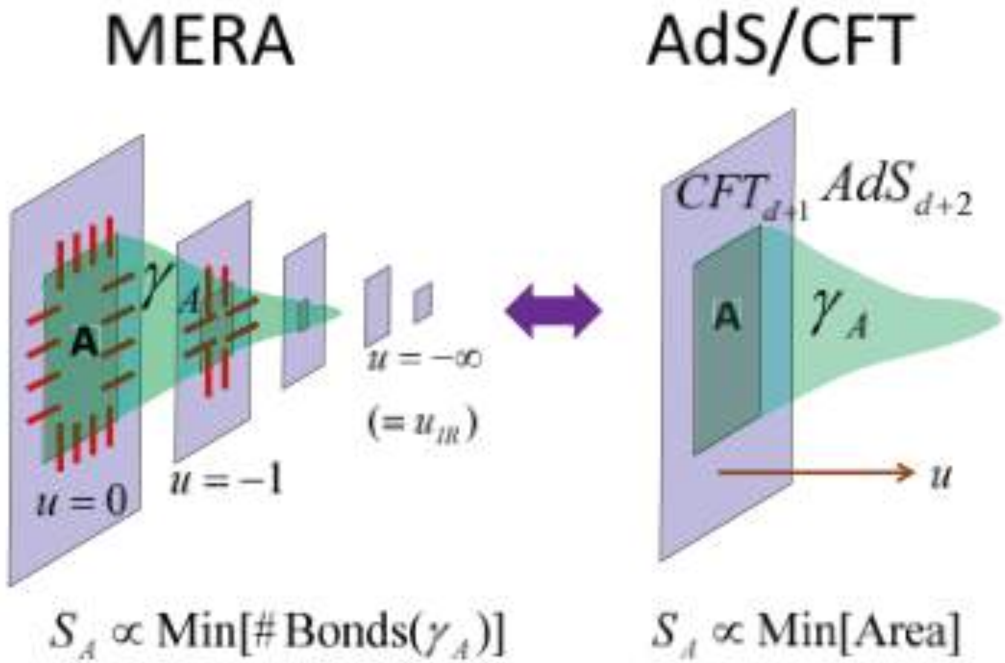
In 2d, ansatz for e.g., Fermi & Bose liquids, $S(L) \approx L \log L$

Increasing complexity...



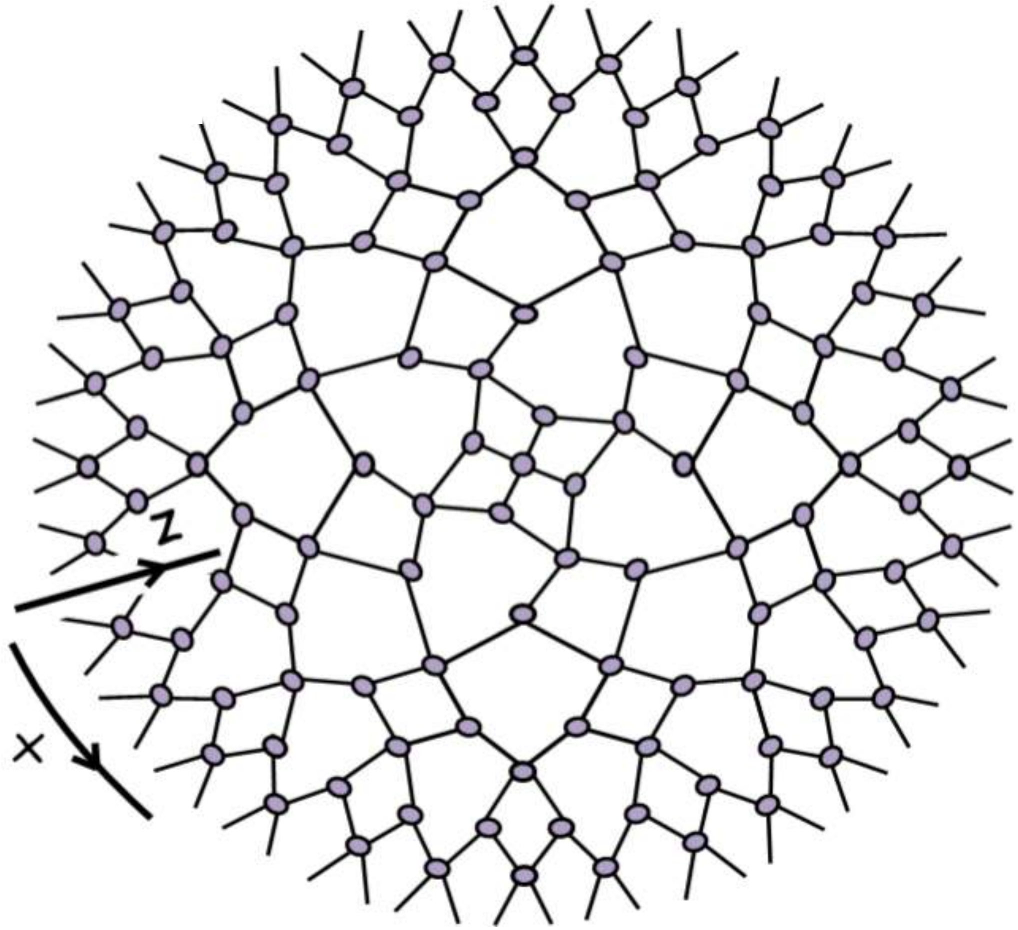
8) MERA & AdS/CFT

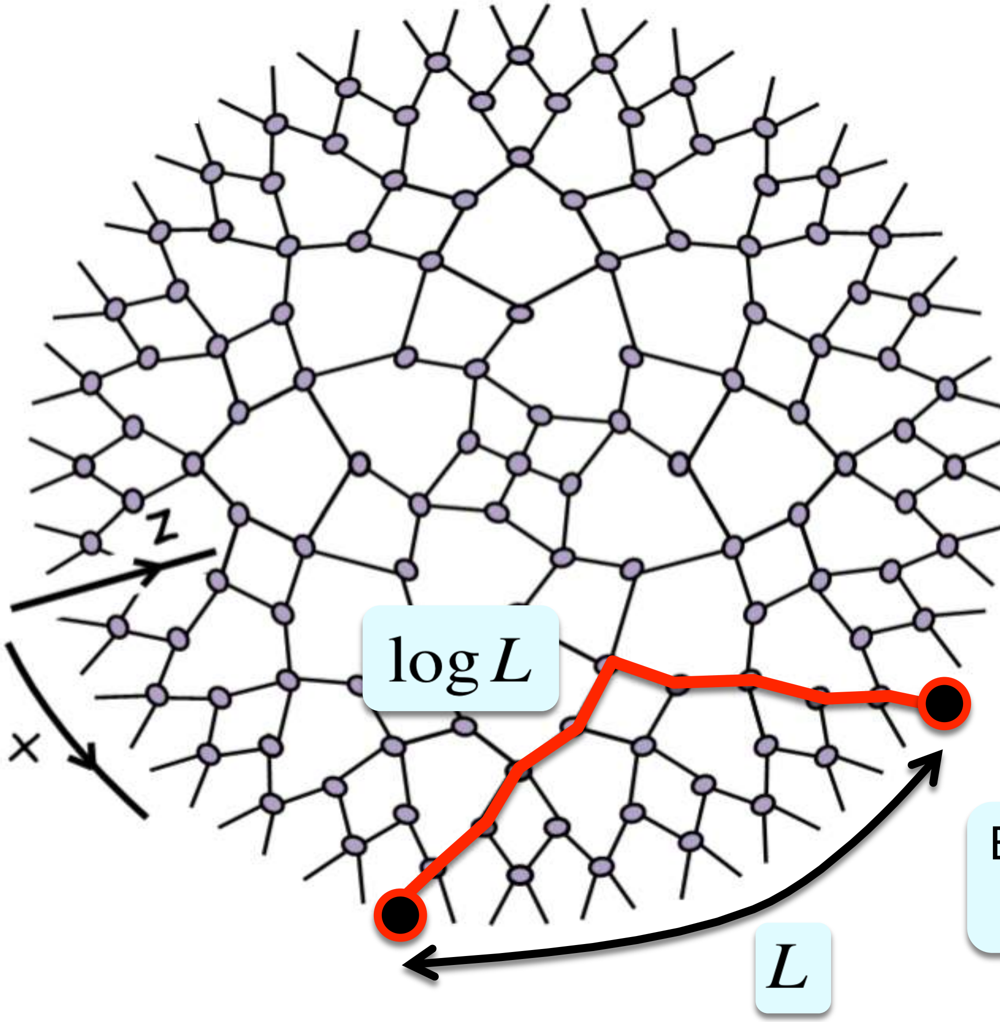
e.g. B. Swingle, PRD 86, 065007 (2012), G. Evenbly,G. Vidal, JSTAT 145:891-918 (2011)



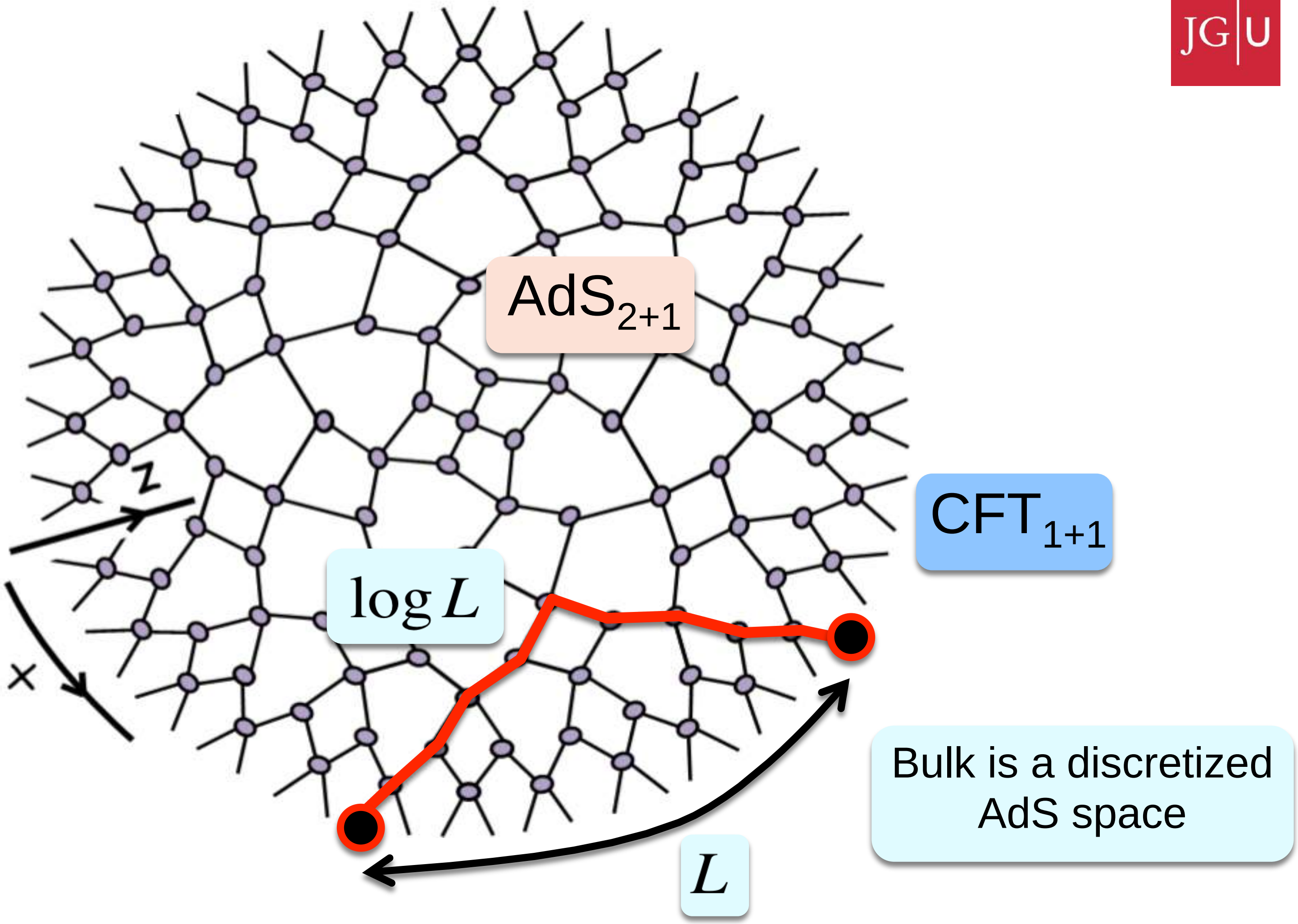
Picture from M. Nozaki, S. Ryu, T. Takayanagi, JHEP10(2012)193

MERA entropy ~ Ryu-Takayanagi prescription

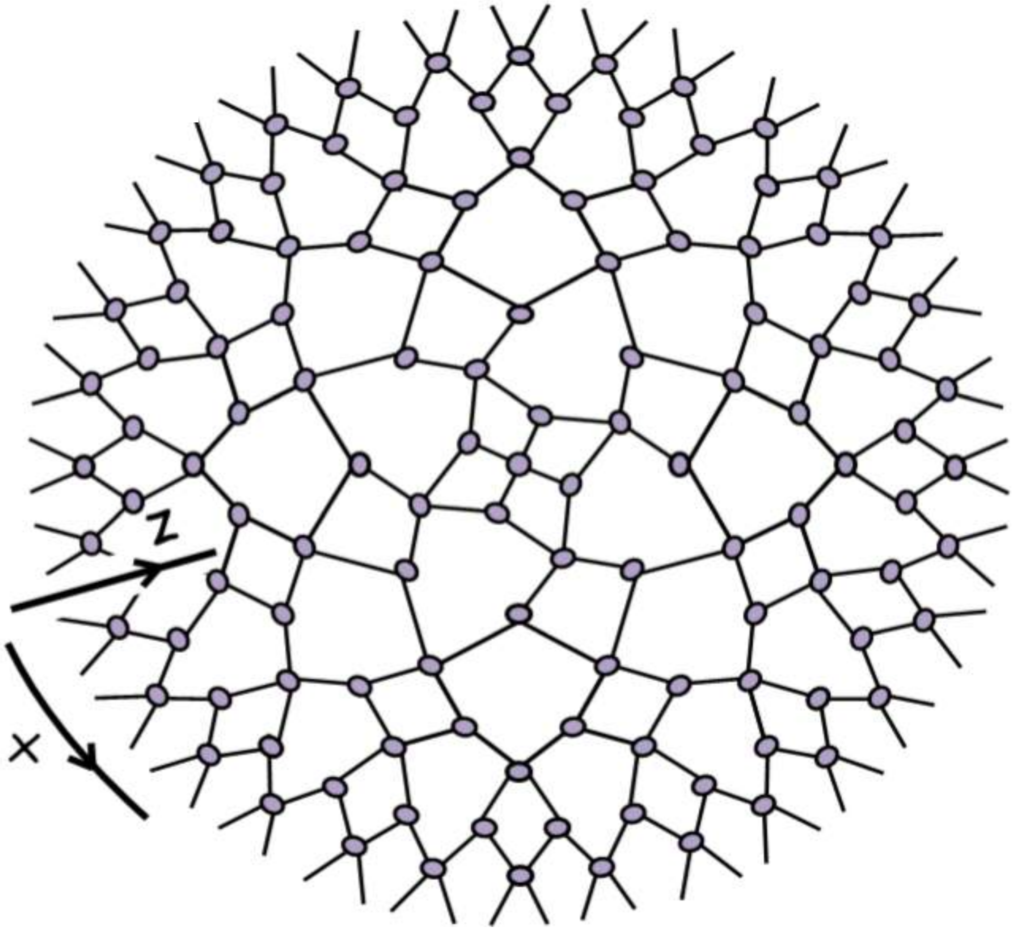

z
x

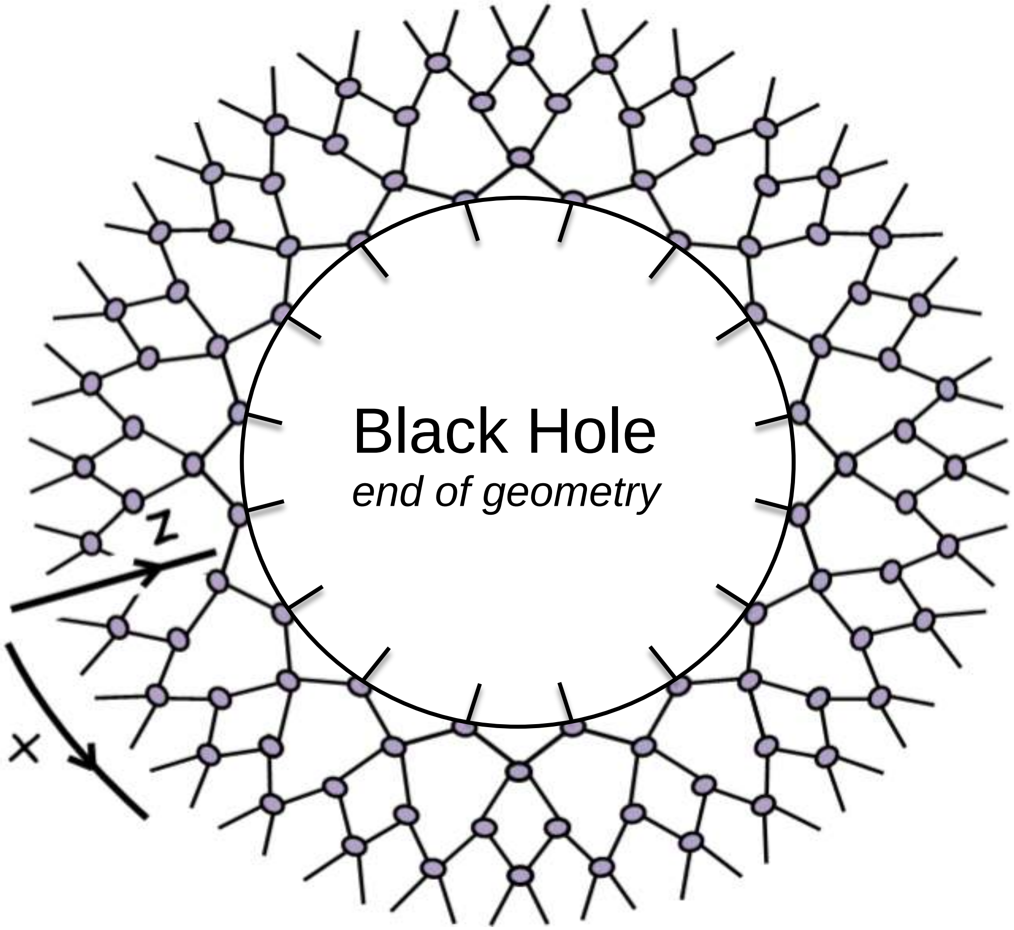


Bulk is a discretized AdS space



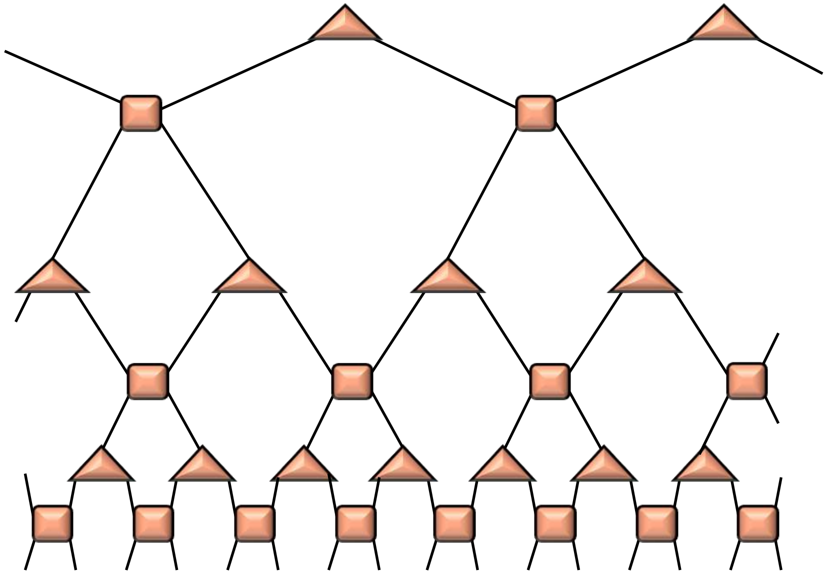
For a scale-invariant MERA, the tensors of a critical model with a CFT limit correspond to a „gravitational“ description in a discretized AdS space:
„lattice“ realization of AdS/CFT correspondance


z
x

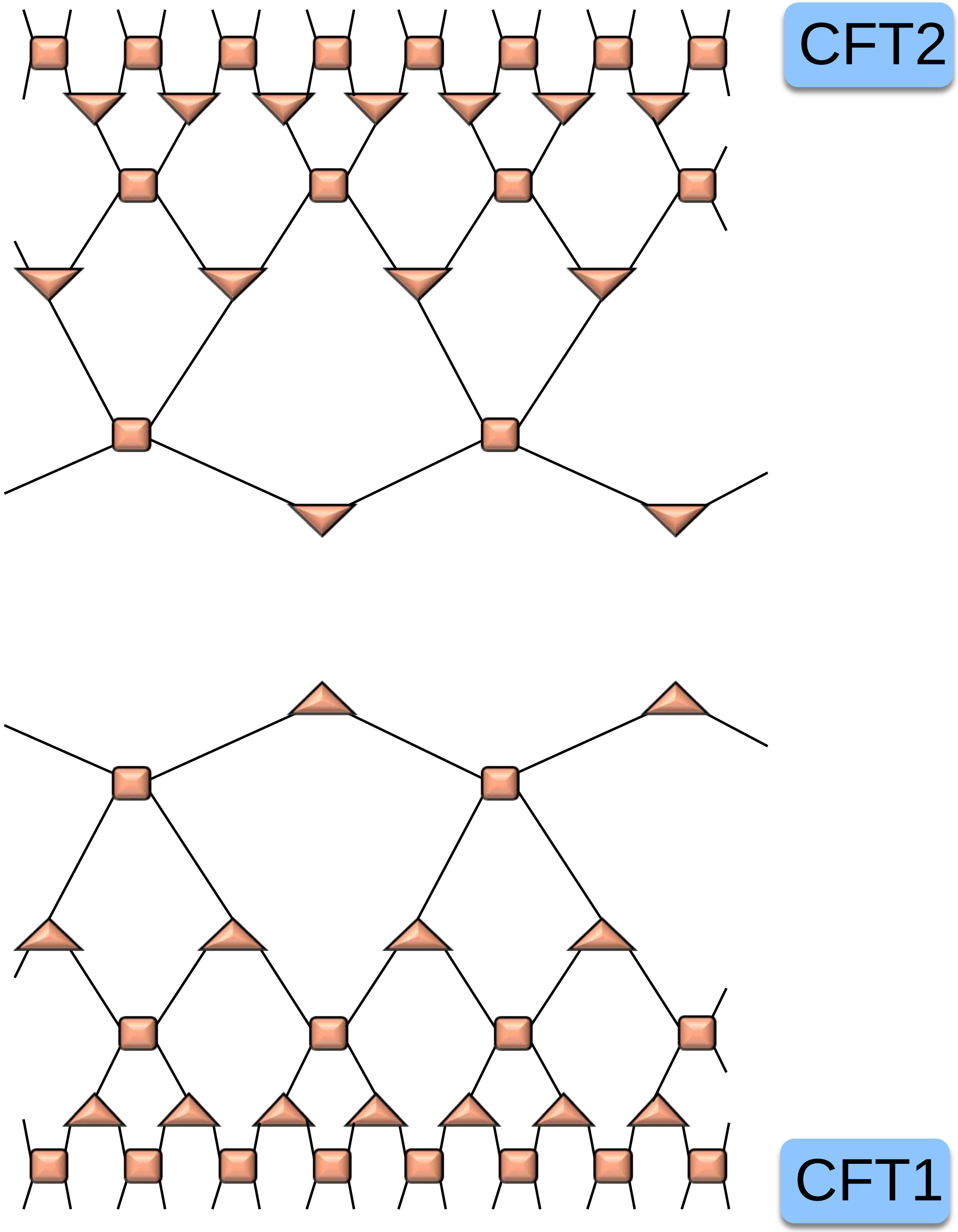


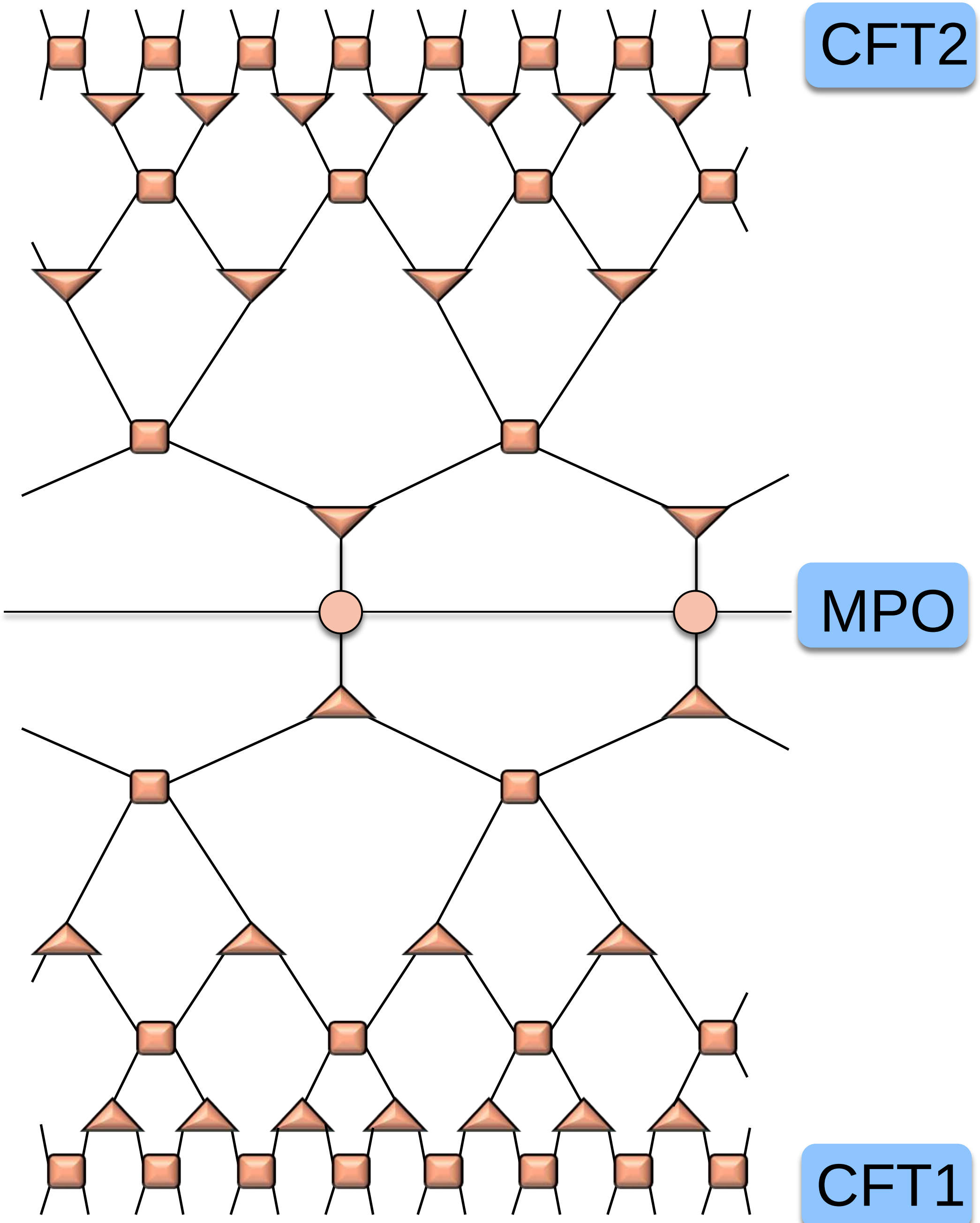
Scale invariance broken by the temperature
~ number of MERA layers

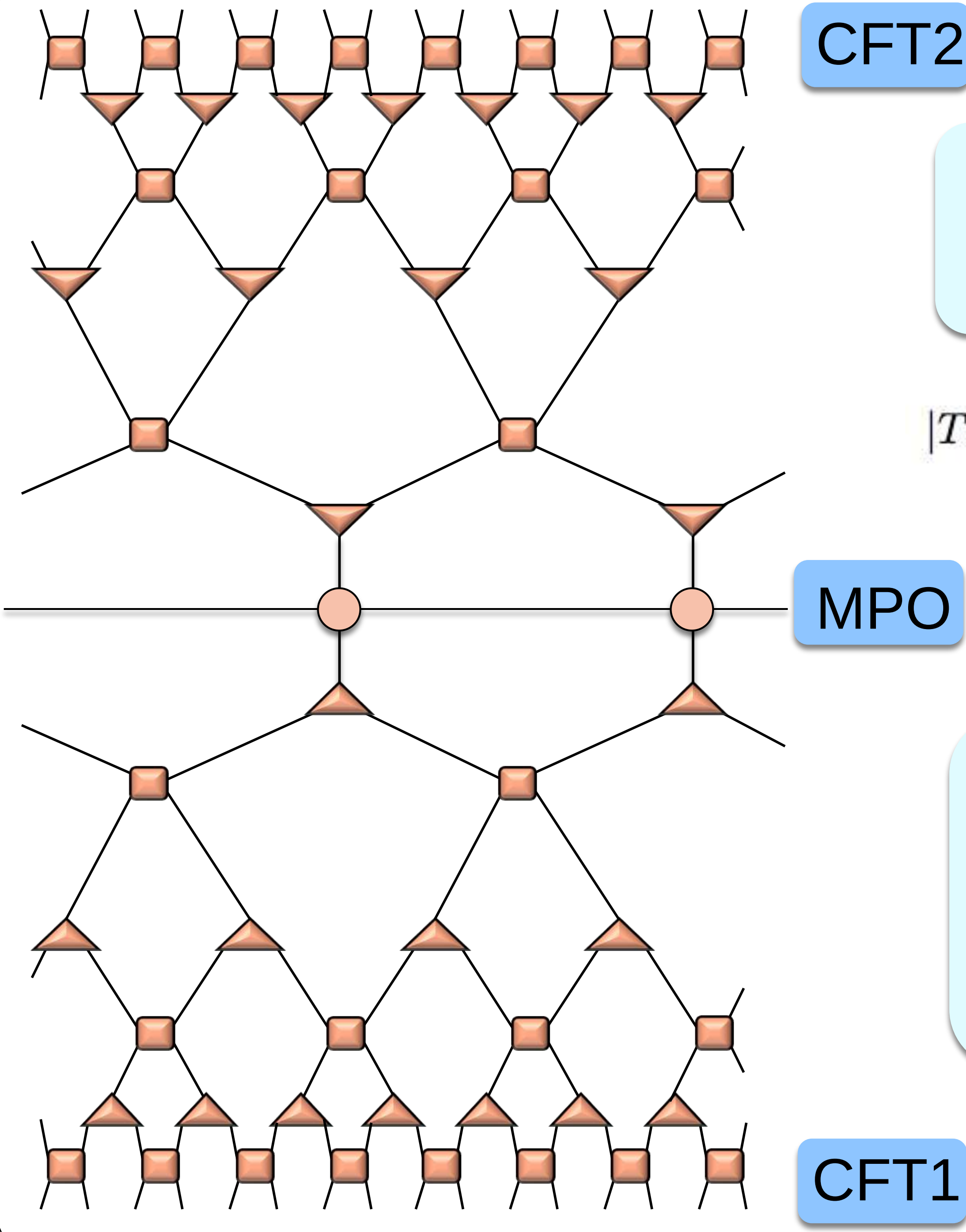
Thermal state = density matrix after tracing out the interior



CFT1


CFT2
CFT1


CFT2
MPO
CFT1



~ wormhole?

Thermofield double state

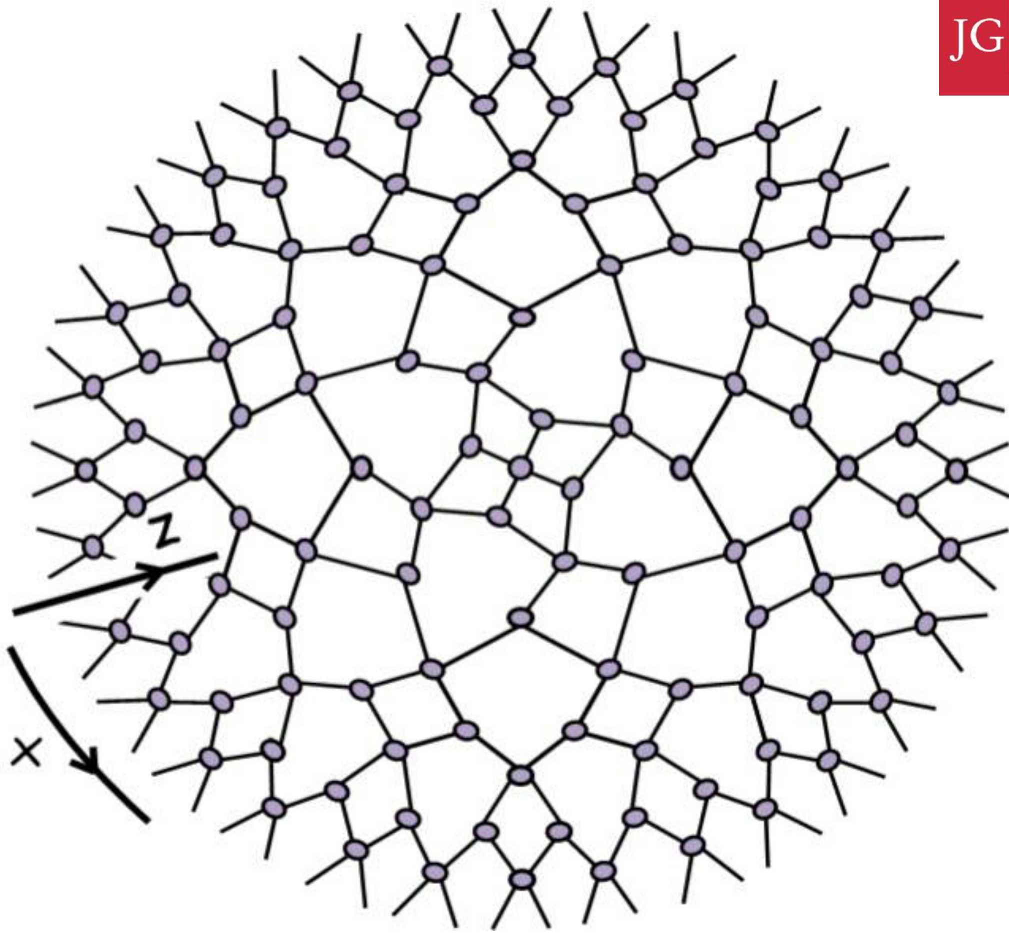
Eternal AdS black-hole

$$|TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |n\rangle_1 |n\rangle_2$$

Entanglement connects upper and lower spacetimes

ER=EPR
Maldacena & Susskind
Van Raamsdonk...

MERA



cMERA

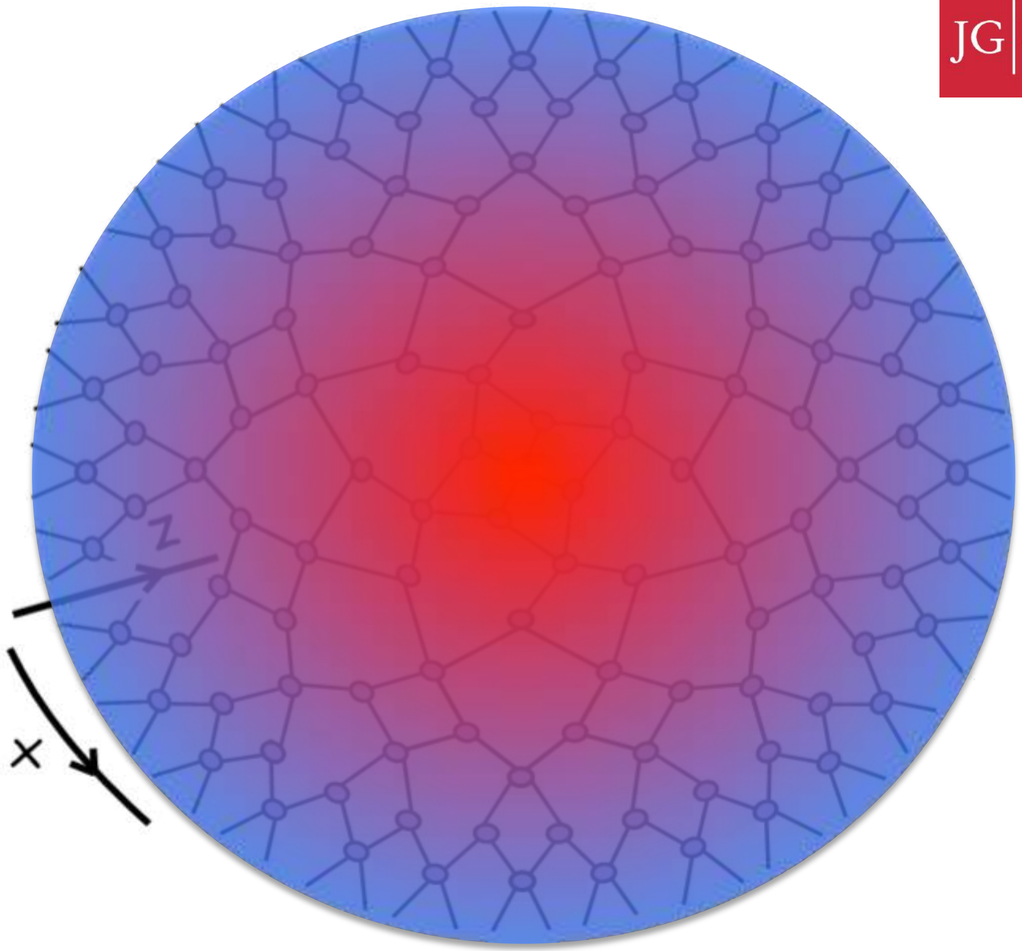
(continuum)

$$|\psi\rangle = P e^{-i\int_{u2}^{u1}\left(K(u)+L\right)du}\,|\Omega\rangle$$

J. Haegeman et al,
Phys. Rev. Lett. 110, 100402 (2013)

$K(u)$ Disentangler generator

L Isommetry generator



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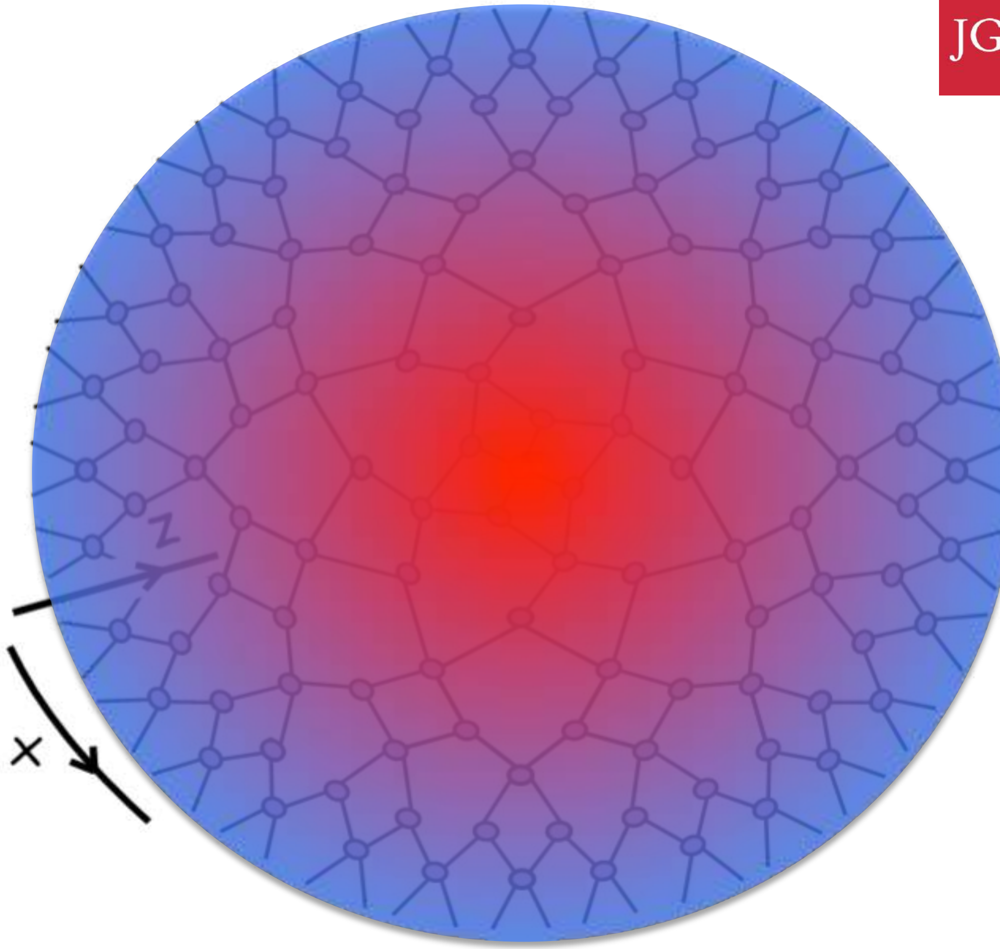
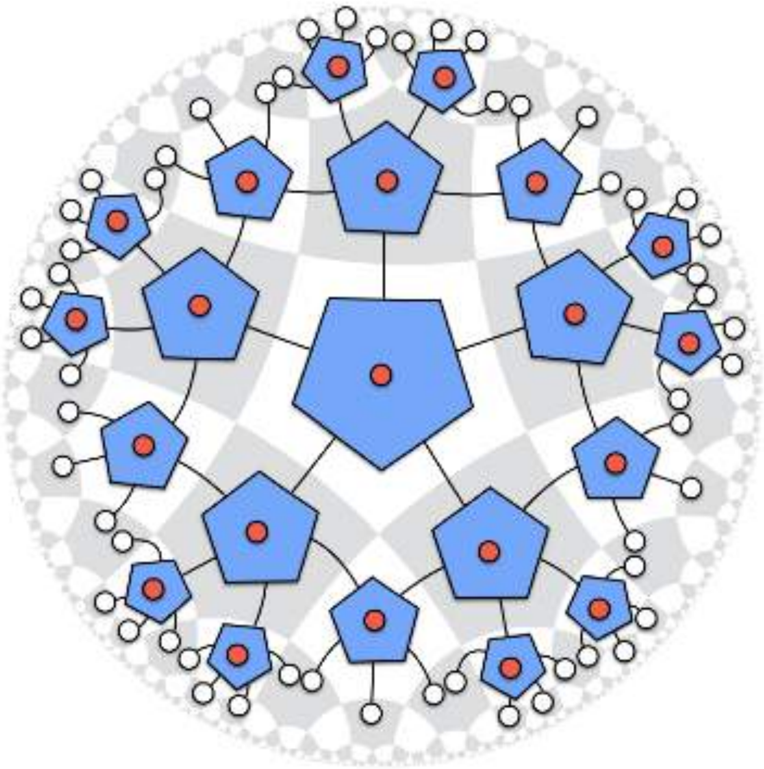


$$g_{uu}(u)du^2 = \mathcal{N}^{-1}\left(1 - \left|\langle\Psi(u)|e^{iL\cdot du}|\Psi(u+du)\rangle\right|^2\right)$$

Measures the density of strength of disentanglers.
Reproduces AdS metric

M. Nozaki, S. Ryu, T. Takayanagi, JHEP10(2012)193



Holographic quantum error-correcting codes

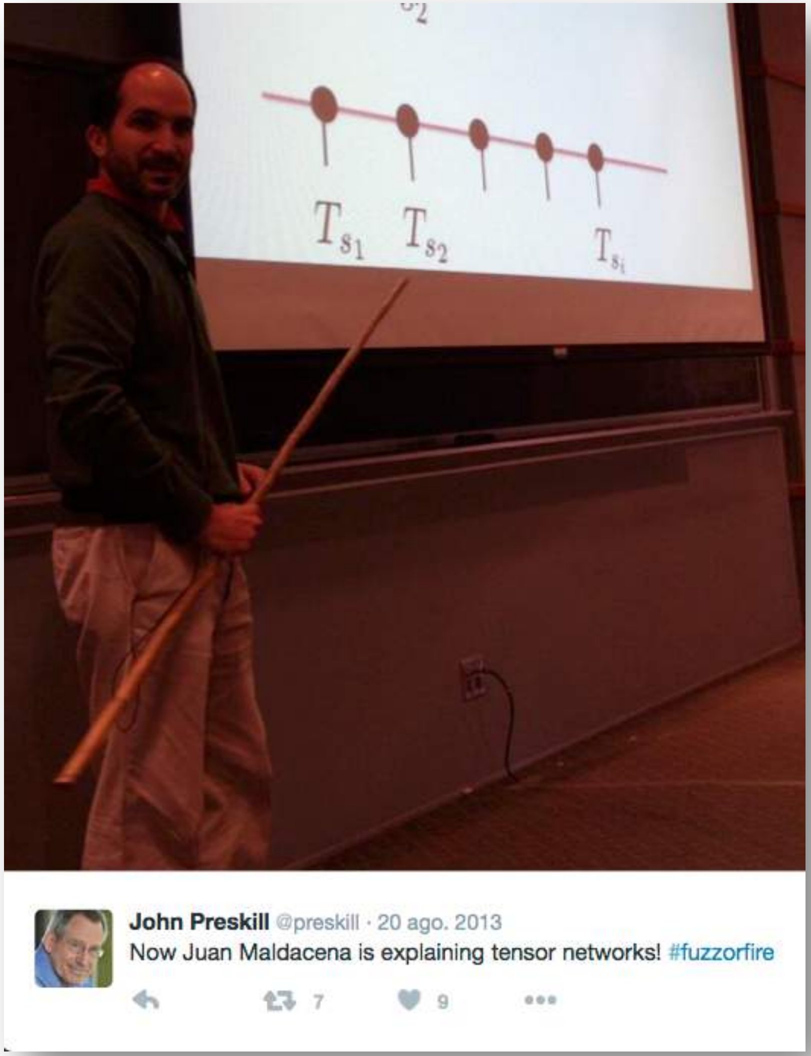
F. Pastawski et al, JHEP 06 (2015) 149

Exact bulk-boundary correspondance

Is AdS/CFT a quantum error-correcting code?

A. Almheiri, Y. Dong, D. Harlow, JHEP 1504:163,2015

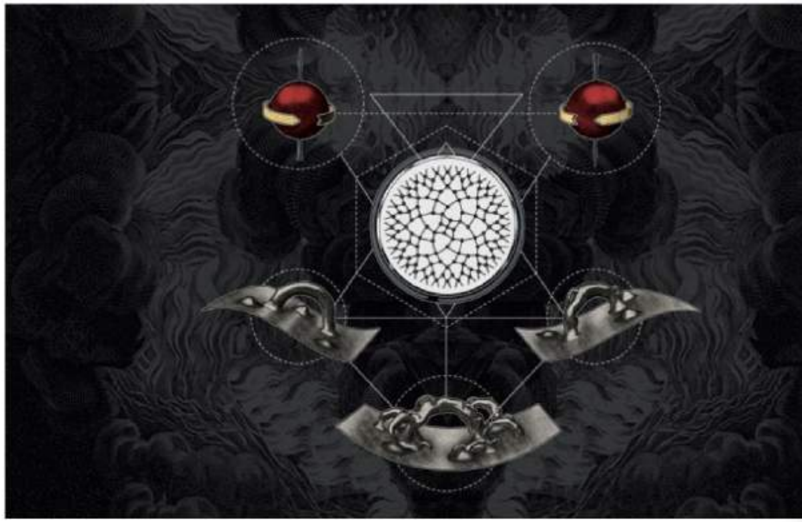
Getting popular even on Twitter…



... and the media ...

Quanta Magazine

Series: **The Quantum Fabric of Space-Time**



How Quantum Pairs Stitch Space-Time

New tools may reveal how quantum information builds the structure of space.

By Jennifer Ouellette

nature International weekly journal of science

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NATURE | NEWS FEATURE

The quantum source of space-time

Many physicists believe that entanglement is the essence of quantum weirdness — and some now suspect that it may also be the essence of space-time geometry.

Ron Cowen

16 November 2015

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BY TOM SIEGFRIED OCTOBER 30, 2015

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CONTEXT QUANTUM PHYSICS, GENERAL RELATIVITY AT 100

Tensor networks get entangled with quantum gravity

BY TOM SIEGFRIED 1:59PM, SEPTEMBER 2, 2014

Some cross-over open questions

- cMERA, Wavelets, and AdS/CFT?
- Consistency conditions for AdS/TN?
- TN structure of, e.g., N=4 SYM? (Type-IIB on $AdS_5 \times S^5$)
  Can one derive string theory from entanglement?
- Non-classical gravity from „exotic“ TNs?
  (topological order & D-branes, TNs with symmetries...)
- Holographic multipartite entanglement?
- Holographic mixed-state entanglement?
- „Gravitational“ interpretation of branching MERA?
- Numerical simulations of gravity with TN methods?
- Connection between „entanglement renormalization“ and „holographic renormalization“?

Thank you!

(another good reason to come back...)

www.romanorus.com/SouthamptonTutorial2016.pdf