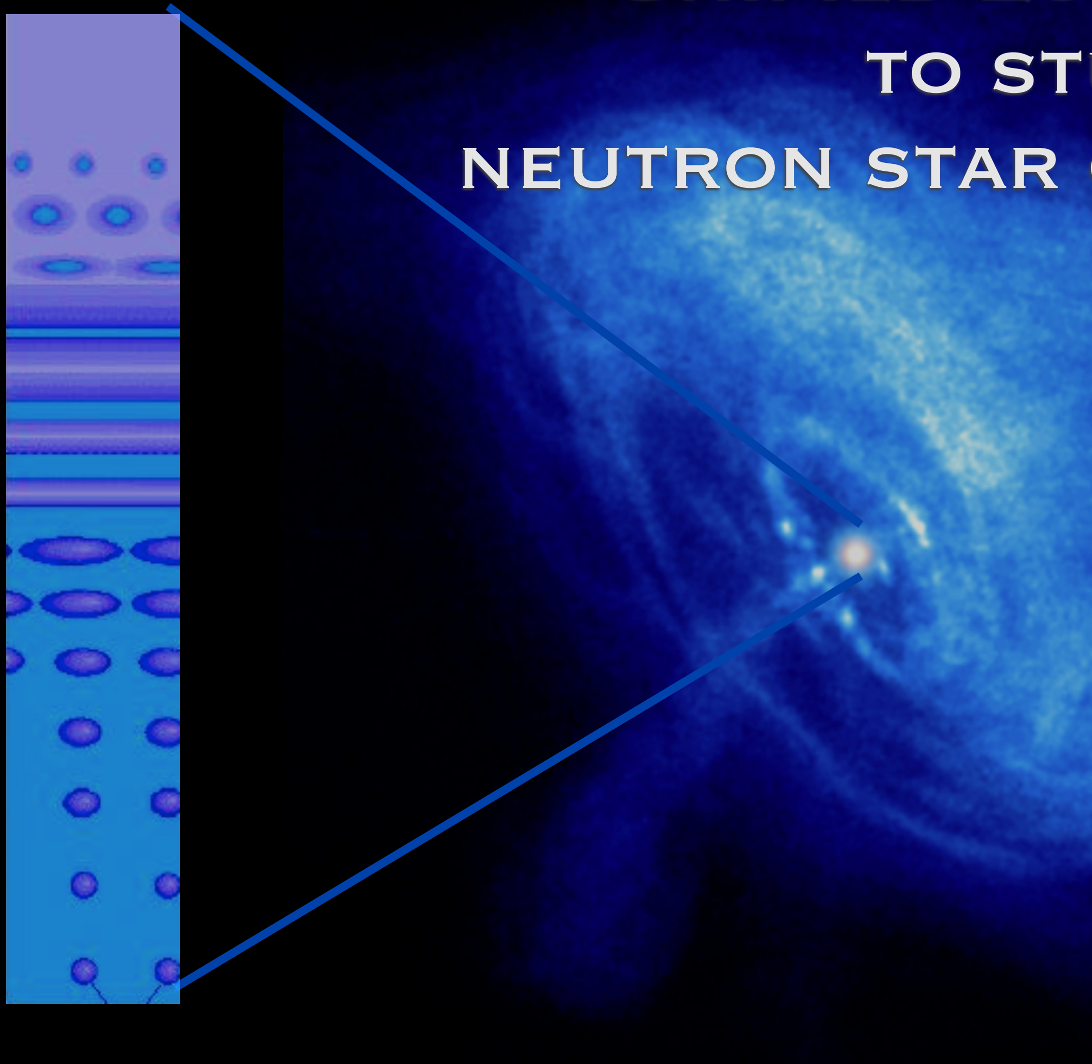
Unified EoS models to study neutron star oscillations



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Collaborators:

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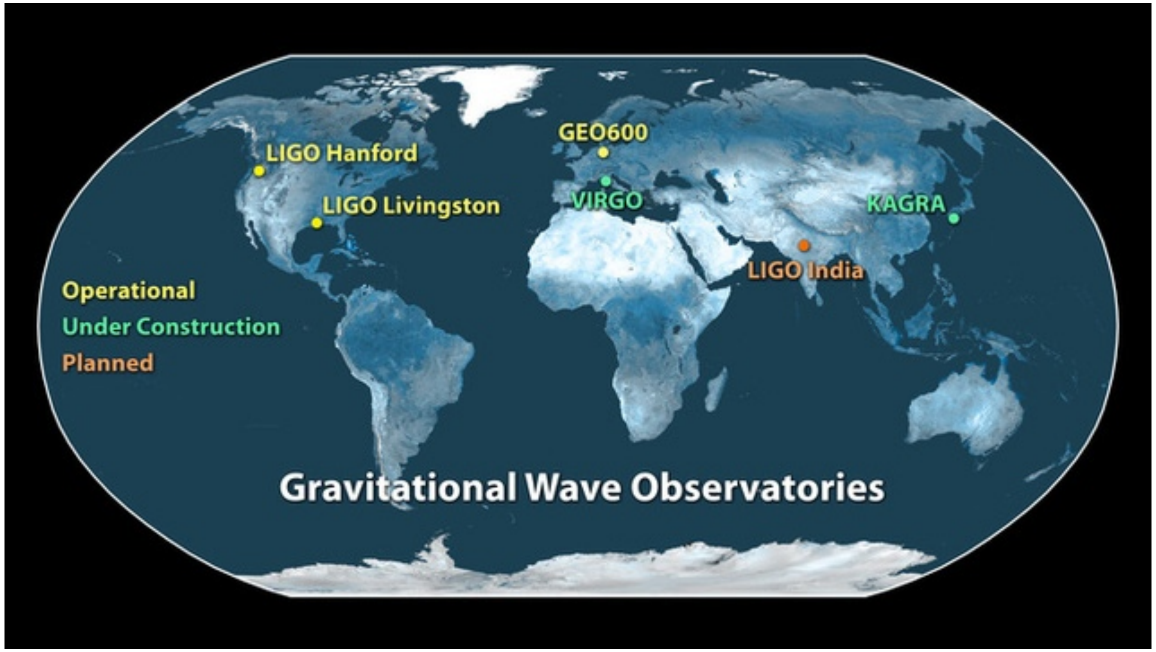
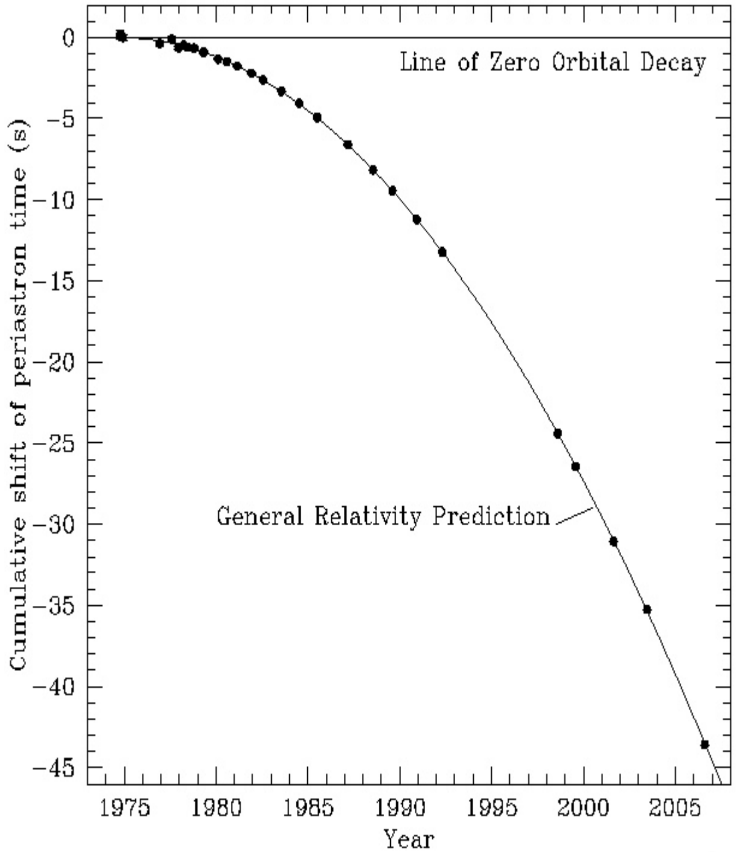
Jerome Margueron

CNRS — dépasser les frontières
lpc caen — Laboratoire de physique corpusculaire
ENSICAEN — École Nationale Supérieure d'Ingénieurs de Caen

NewCompStar W.G. Meeting,
Southampton, U.K.,
13 Sept 2016

Gravitational Wave asteroseismology



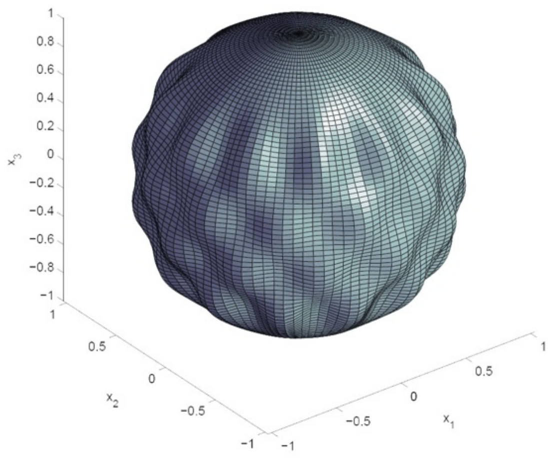


PSR B1913+16

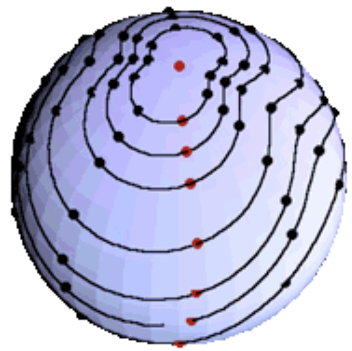
G W detectors

Gravitational Waves from Neutron Stars

p-mode



R-mode



- Mountains on the surface
- Strong magnetic fields
- Non-axisymmetric Oscillations:

  f-modes: fundamental

  g-modes: buoyancy

  p-modes: pressure

  w-modes: space-time

  R-modes: Coriolis force

Gravitational Waves from Neutron Stars

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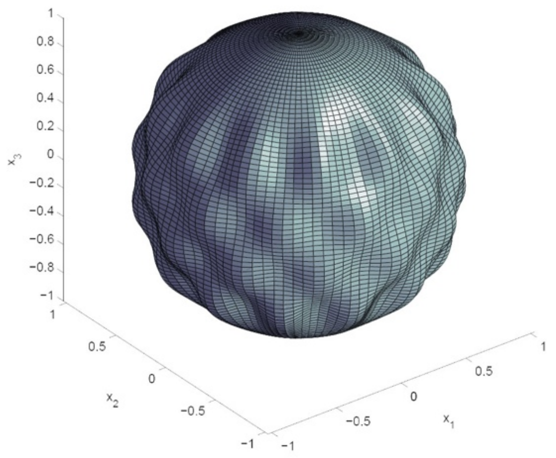
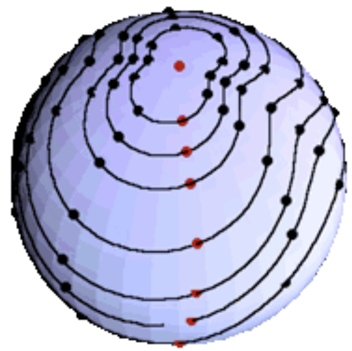


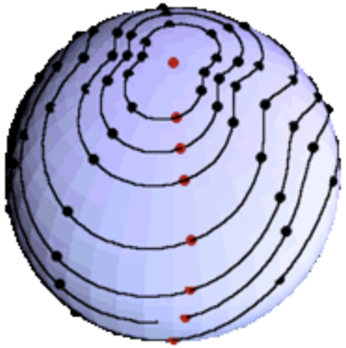
R-mode



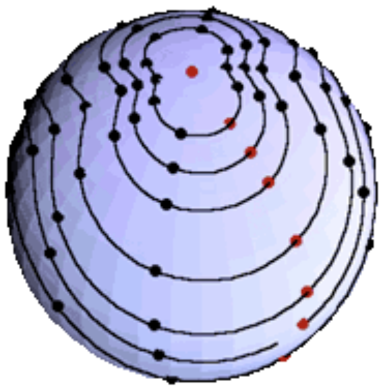
- Mountains on the surface → elasticity of the crust
- Strong magnetic fields
- Non-axisymmetric Oscillations:
  - f-modes: fundamental
  - g-modes: buoyancy
  - p-modes: pressure
  - w-modes: space-time
  - R-modes: Coriolis force

→ oscillation modes exist in the solid crust, liquid core and the interfaces of the different regions of the NS

R-modes: probe of NS interior



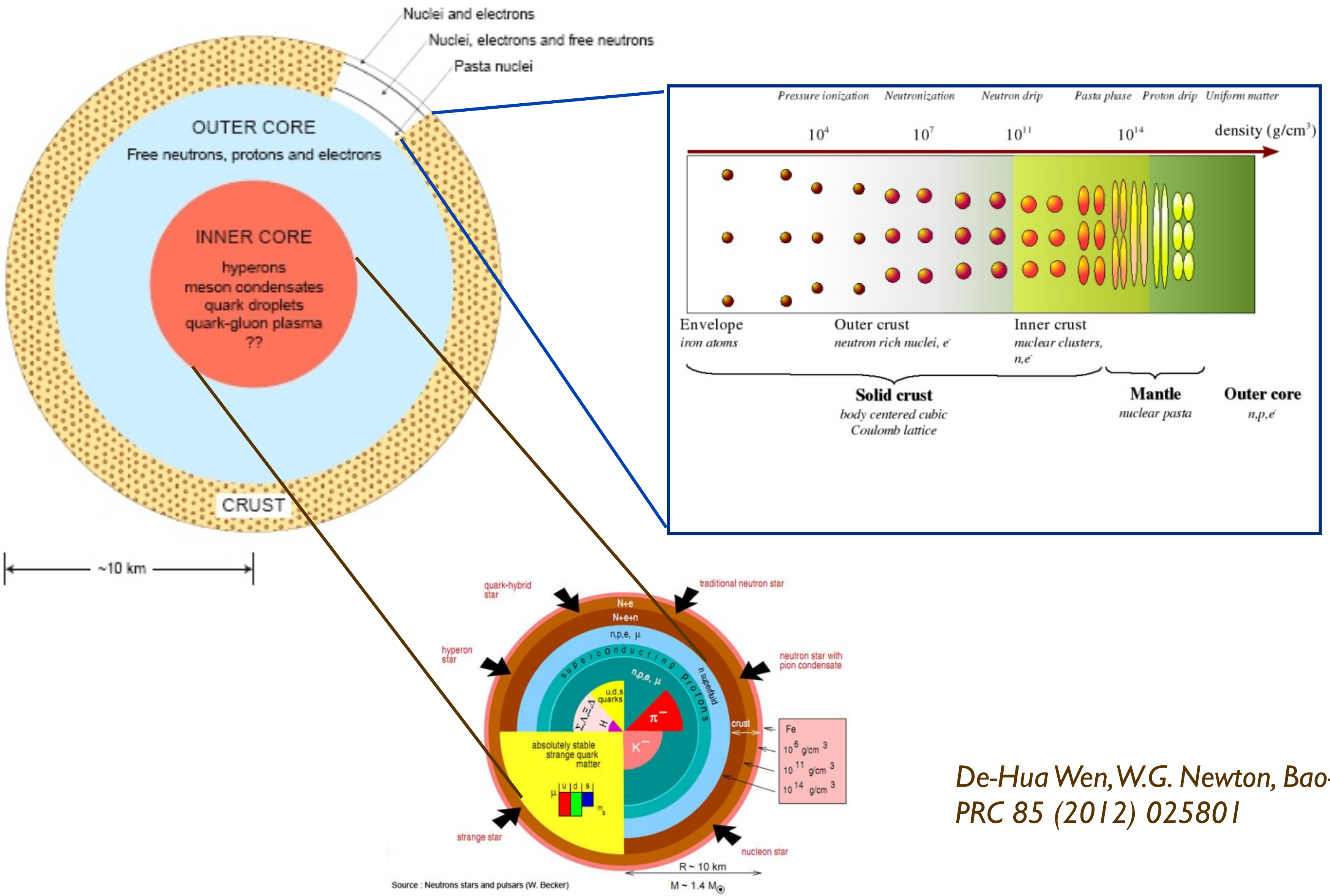
co-rotating



inertial

* *R-modes are generic to all rotating neutron stars*
* *They are unstable by the CFS mechanism: R- mode amplitude grows under the effect of its own gravitational radiation-reaction; possible sources of GW. They can spin down newborn rapidly-rotating NSs, or limit the spin-up of accretion powered MSPs in LMXBs*
* *The instability can be damped by (shear, bulk) viscosity, which depend on the composition of the neutron star interior*
* *Shear viscosity results from momentum transport due to particle scattering in the crust; For normal fluids, the main contribution comes from n-n scattering; bulk viscosity results from variation in pressure and density in the core when the system is driven away from beta equilibrium*
* *For older neutron stars, one of the main damping mechanisms comes from the viscous boundary layer at the crust-core interface*
* *timescale associated with growth/dissipation*
  $\tau_{\zeta,\eta} \gg \tau_{GW}$ *: r-mode unstable, star spins down*
  $\tau_{\zeta,\eta} \ll \tau_{GW}$ *: r-mode damped, star can spin rapidly*

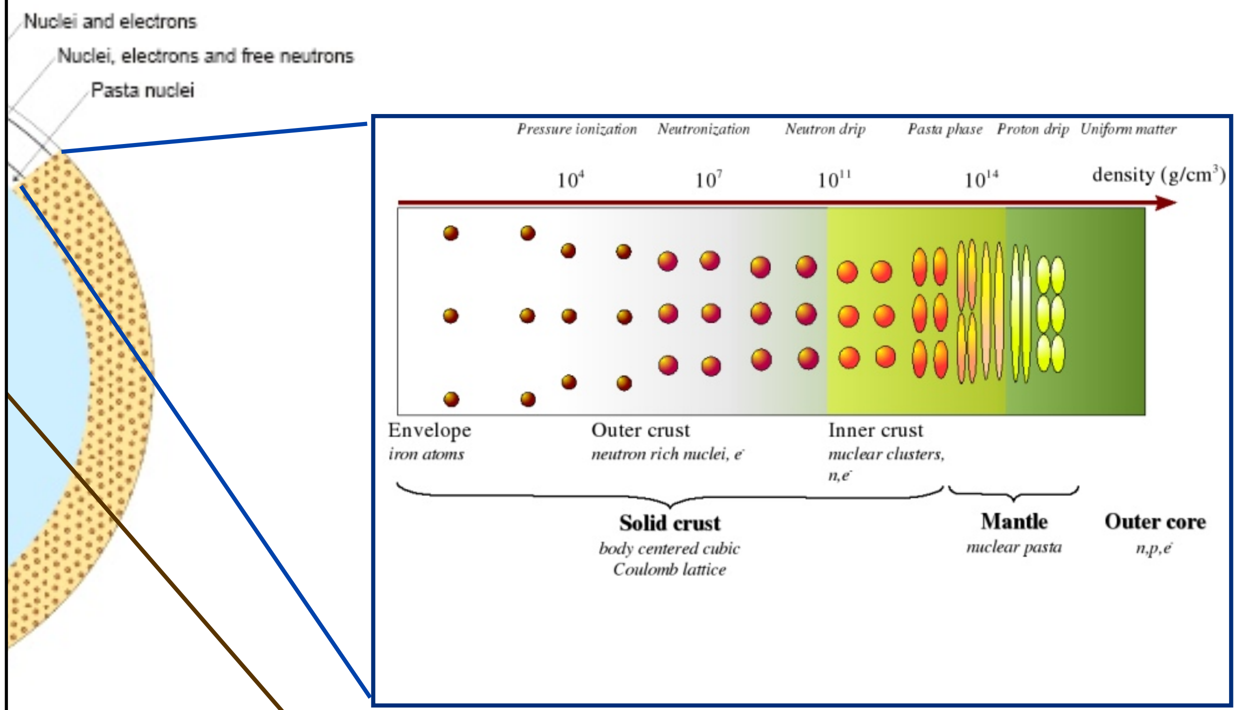
CRUST-CORE TRANSITION AND R-MODES

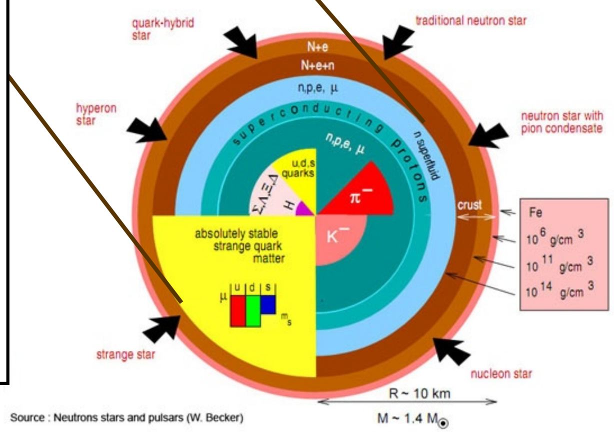


De-Hua Wen, W.G. Newton, Bao-An Li, PRC 85 (2012) 025801

CRUST-CORE TRANSITION AND R-MODES

- *Studies of r-mode instability focus on a canonical NS described by a polytropic EoS*
- *For older NSs, simple models with a rigid crust overestimate the dissipation at the viscous boundary layer, which is sensitively dependent on the crust thickness (depends on R and crust-core transition density)*
- *Crustal thickness is not calculated consistently: (usually assume canonical value 1.5 x 10^{14} g cm^{-3}, or varied independently of the core)*





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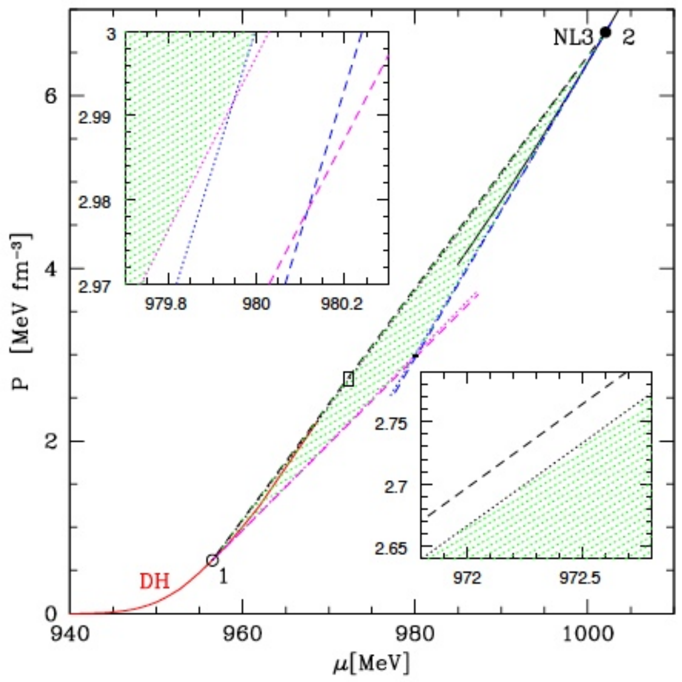
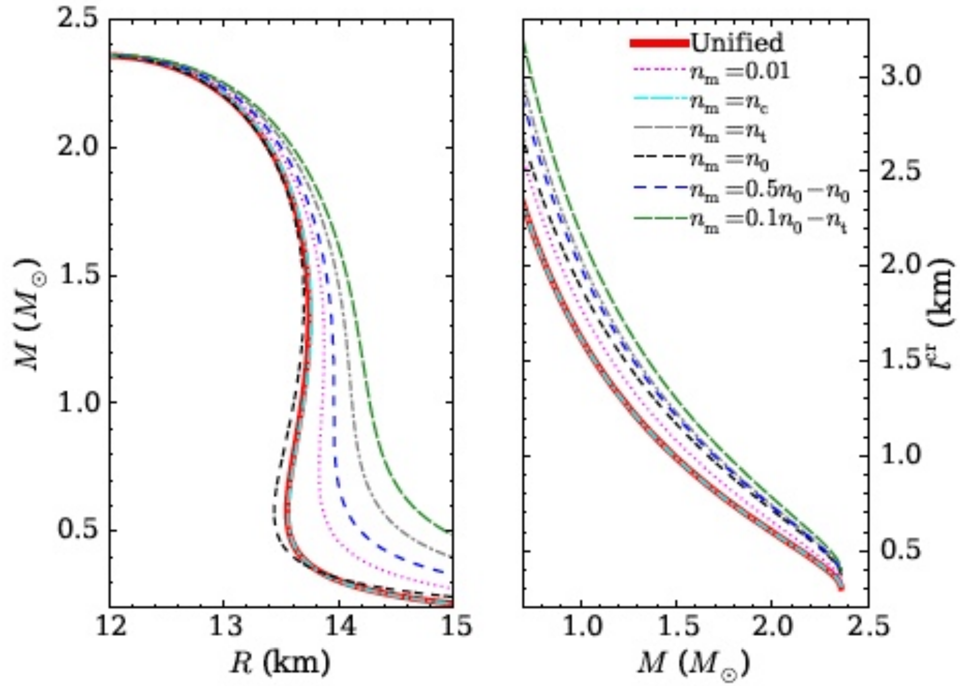
WHY WE NEED A UNIFIED EOS



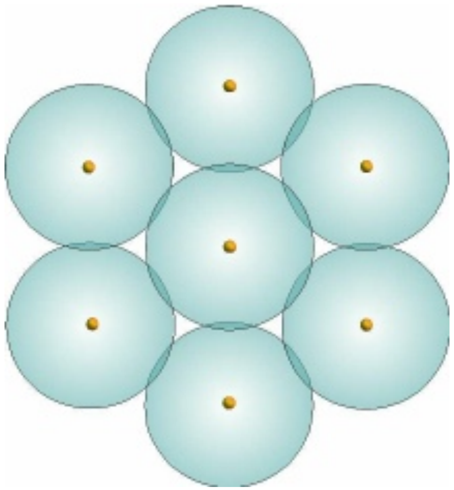
Fortin et al., ArXiv: 2016

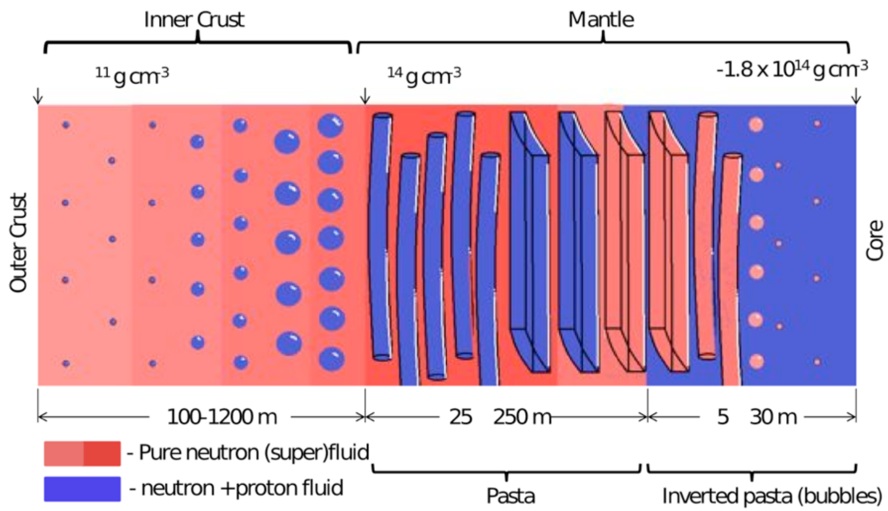
Crust-Core Matching for non-unified EoSs

- Matching is done so that pressure is an increasing function of energy density
- different models leads to arbitrary results
- uncertainty in crust thickness upto 30% and for radius 4%



MODELING NS CRUST & CORE





Outer crust

- *nuclei in lattice + e gas*
- *each lattice volume represented by a Wigner-Seitz cell*
- *Each cell assumed to be charge neutral, Coulomb interaction between cells neglected*
- *electrons uniformly distributed*
- *system in chemical equilibrium*
- *Composition largely sensitive to experimental determination of nuclear masses [Audi tables]*

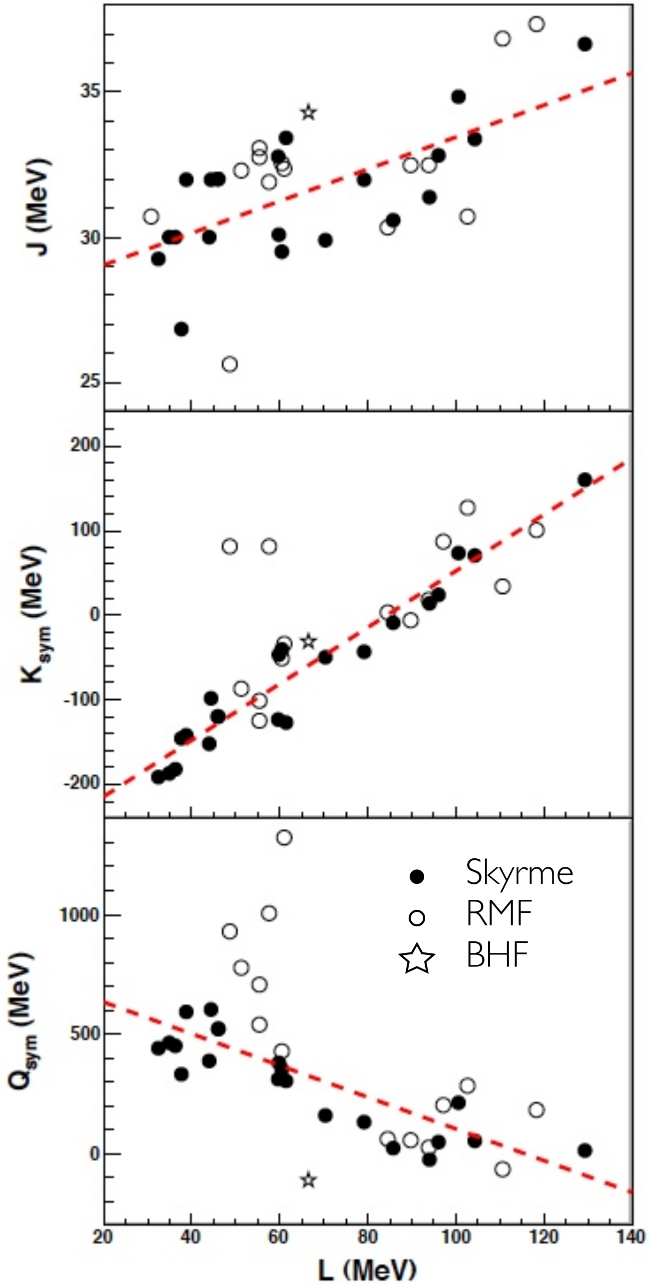
Inner crust

- *as density increases, nuclei become n rich*
- *Beyond density $4x10^{11}$ g/cm^3 (n drip density), n drip out of nuclei*
- *nuclear clusters + e gas + n gas*
- *At T=0, ground state energy: E minimization*
- *Composition dominated by clusters beyond current experimental data*

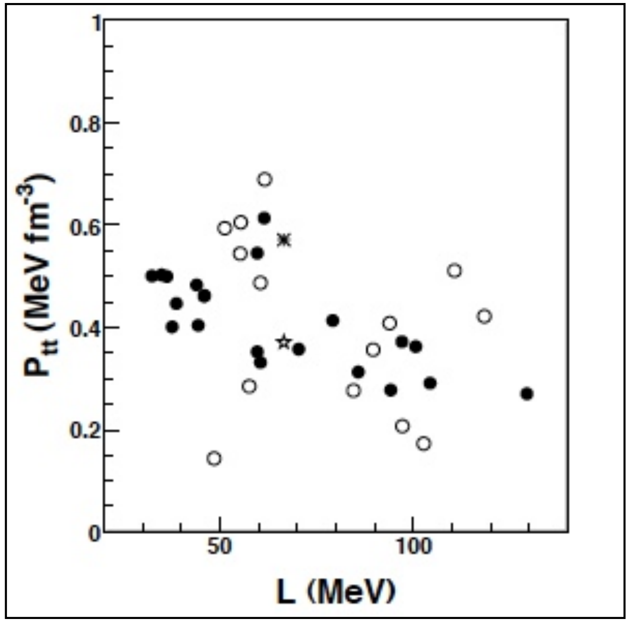
Core

- *Homogeneous liquid composed of n,p,e (exotic matter) in chemical equilibrium*

WHY WE NEED A MODEL INDEPENDENT EOS

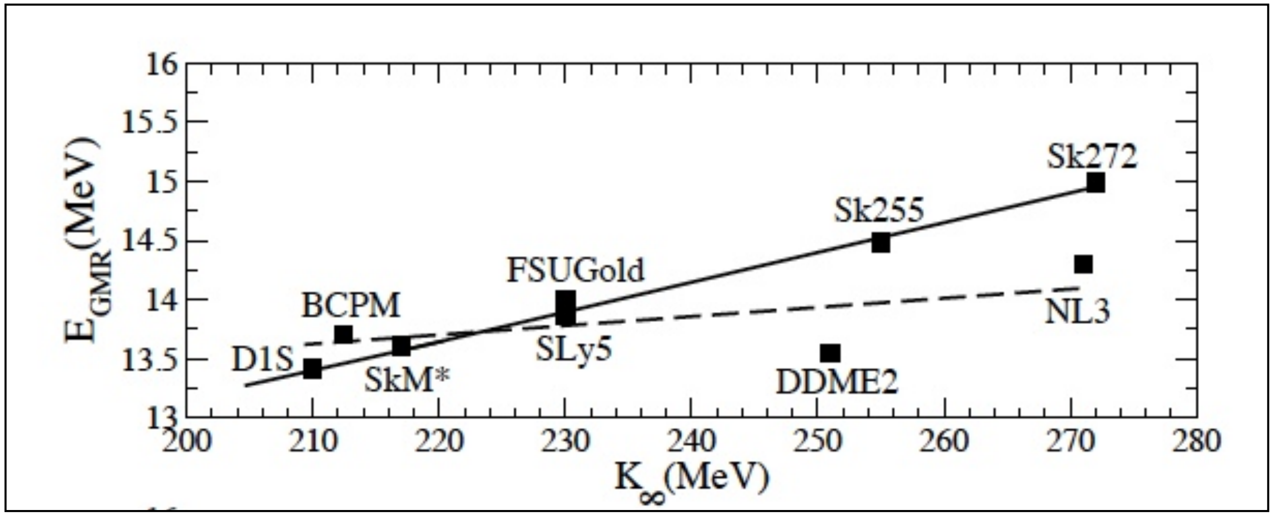


Ducoin et al. PRC 2011



Spurious correlations

Khan Margueron PRC 2013



TOWARDS A MODEL INDEPENDENT UNIFIED EOS

- *to study of the properties of NS crust-core transition, it is crucial to construct a unified consistent scheme, to describe both the homogeneous nuclear matter in the core and the clusterized matter in the crust*

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- *NS crust with subnuclear density*
  $n < 0.16\ fm^{-3}$
  empirical constraints from terrestrial nuclear physics
  => a model-independent EoS, based on empirical constraints

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Nuclear matter model (ab-initio, pheno, empirical)

$$\delta = \frac{\rho_n - \rho_p}{\rho} \qquad x = \frac{\rho - \rho_0}{\rho_0}$$

$$\begin{aligned} e(\rho_n, \rho_p) &= e(\rho, \delta) \\ &= e_{IS}(\rho) + e_{IV}(\rho)\delta^2 + O(\delta^3) \\ &= \left(\boldsymbol{E_0} + \frac{1}{18}\boldsymbol{K_0}x^2 + O(x^3)\right) + \left(\boldsymbol{J_0} + \frac{1}{3}\boldsymbol{L}x + \frac{1}{18}\boldsymbol{K_{sym}}x^2 + O(x^3)\right)\delta^2 \end{aligned}$$

- Around saturation: If $(E_0, \rho_0, K_0, J_0, L, K_{sym})$ are known, the EoS is known
- Above saturation: it is the same, but you need more coefficients

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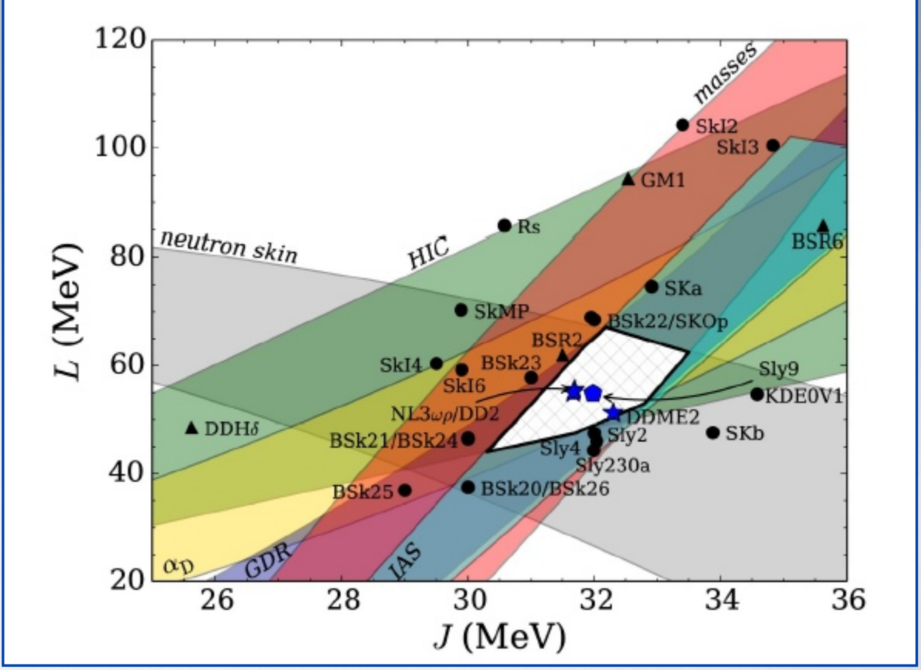
Constraints in J-L plane

- From n skin thickness of ^{208} Pb
- From HIC
- From electric dipole polarizability α_D
- From giant dipole resonance (GDR) of ^{208} Pb
- From measured nuclear masses
- From isobaric analog states (IAS)



Fortin et al., ArXiv: 2016

PRESENT UNCERTAINTY IN EMPIRICAL PARAMETERS

Margueron, Casali, Gulminelli (in preparation)

		fixed		Explore inside small interval					Consider large interval		
Model		ρ_0	E_0	K_0	Q_0	Z_0	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}
		fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV
Skyrme	Average	0.1586	-15.91	251.68	-300.20	1178.35	31.22	53.52	-130.15	316.68	-1890.99
	σ	0.0040	0.21	45.42	157.81	848.47	2.03	31.06	132.03	218.23	1191.23
RMF	Average	0.1494	-16.24	267.99	-1.94	5058.30	35.11	90.20	-4.58	271.07	-3671.83
	σ	0.0025	0.06	33.52	392.51	2294.07	2.63	29.56	87.66	357.13	1582.34
RHF	Average	0.1540	-15.97	248.06	389.17	5269.07	33.97	90.03	128.16	523.29	-9955.49
	σ	0.0035	0.08	11.63	350.44	838.41	1.37	11.06	51.11	236.80	4155.74
Average		0.1540	-16.04	255.91	29.01	3835.24	33.43	77.92	-2.19	370.34	-5172.77
σ		0.0051	0.20	34.39	424.59	2401.14	2.64	30.84	142.71	298.54	4362.35

$$\frac{E}{A} = (E_0 + E_{sym}\delta^2) + L_{sym}x\delta^2 + \frac{1}{2}(K_0 + K_{sym}\delta^2)x^2 + ...$$

A Unified Model

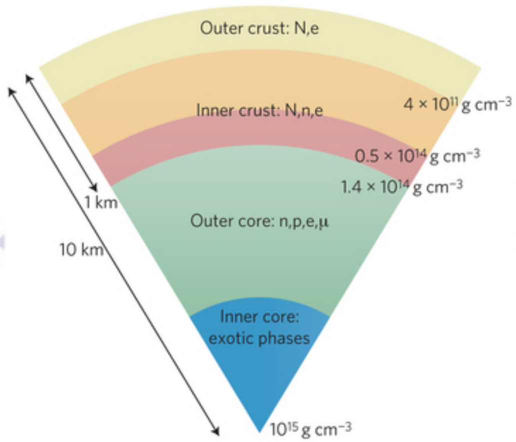
- In density functional theory, the energy density functional

$$\mathcal{H} = \mathcal{H}\left[\rho_q(\vec{r}), \nabla^k_{q'}\rho_q(\vec{r})\right]$$

  is a function of the densities ρ_q and their gradients.
- In general, there are an infinite number of gradients
- If $k = 0 \implies$ Thomas-Fermi approximation (density terms only): applicable only to homogeneous infinite nuclear matter
- For finite nuclei (Inhomogeneous matter), non-zero k (say 2) $\implies$ Extended-Thomas-Fermi approximation of order k
- Separated into isoscalar and isovector parts

$$\mathcal{H}_{ETF} = \mathcal{H}_{IS} + \mathcal{H}_{IV}\delta^2$$

  where the isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$



A Unified Model

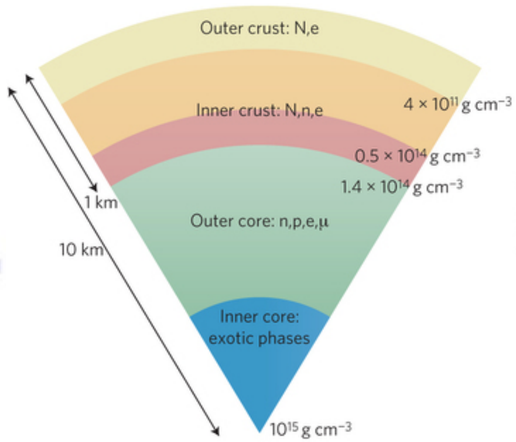
- Each of these contributions consists of kinetic and potential terms

$$\mathcal{H}_{IS,IV} = \mathcal{K} + \mathcal{V}$$

where the potential part can be further separated into contributions from the density-dependent term, the finite range term and the spin-orbit term respectively

$$\mathcal{V} = \mathcal{H}_{\rho} + \mathcal{H}_{fin} + \mathcal{H}_{so}$$

- $\mathcal{K}$ and $\mathcal{H}_{\rho}$ can be determined using empirical constraints
- $\mathcal{H}_{fin}$ and $\mathcal{H}_{so}$ require additional experimental inputs C_{fin} and C_{so}



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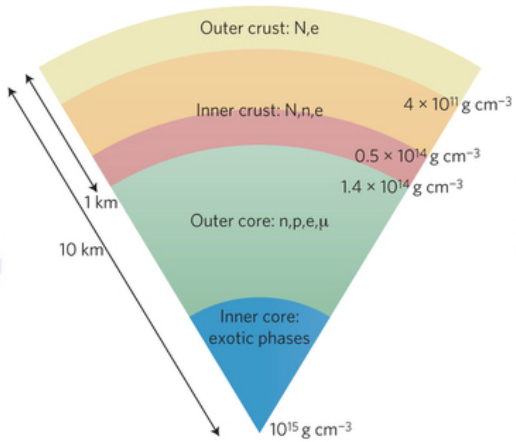

Homogeneous Nuclear Matter (HNM)

- For infinite homogeneous matter, the finite size effects and spin-orbit terms can be neglected
- Energy per particle $e = \mathcal{H}/\rho$ for HNM : expansion around saturation density ρ_0 in terms of $x = \frac{\rho - \rho_0}{3\rho_0}$

$$e(x, \delta) = e_{kin}(x, \delta) + e_{pot}(x, \delta)$$

- The kinetic part in terms of Fermi gas kinetic energy t_0^{FG}

$$e_{kin}(x, \delta) = \frac{1}{2} t_0^{FG} (1 + 3x)^{2/3} \left[(1 + \delta)^{5/3} \frac{m}{m_n^*} + (1 - \delta)^{5/3} \frac{m}{m_p^*} \right]$$

- while the potential part can be written as an expansion

$$e_{pot}(x, \delta) = \sum_{\alpha=0}^{2} (a_{\alpha 0} + a_{\alpha 2} \delta^2) \frac{x^\alpha}{\alpha!}$$

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An Empirical EoS

- The kinetic and potential terms can be regrouped in terms of isospin asymmetry

$$\frac{E}{A} = e = e_{IS} + e_{IV}\delta^2 + \mathcal{O}(\delta^4) \quad (1)$$

- The density dependence of SNM can be expanded around ρ_0, in terms of $x = \frac{\rho - \rho_0}{3\rho_0}$

$$e_{IS} = E_0 + \frac{K_0}{2}x^2 + \mathcal{O}(x^3) \quad (2)$$

where $K_0 = \frac{\partial^2 e}{\partial x^2}\Big|_{x=0,\delta=0}$

An Empirical EoS

- Similarly, symmetry energy around saturation can be written as
$$e_{IV} = J_{sym} + L_{sym}x + \frac{K_{sym}}{2}x^2 + \mathcal{O}(x^3) \qquad (3)$$
where
symmetry energy : $J_{sym} = e_{IV}(x=0) = \frac{1}{2}\frac{\partial^2 e}{\partial \delta^2}\Big|_{\delta=0}$
slope of the symmetry energy : $L_{sym} = \frac{\partial J_{sym}}{\partial x}\Big|_{x=0}$
curvature of symmetry energy : $K_{sym} = \frac{\partial^2 J_{sym}}{\partial x^2}\Big|_{x=0}$
- In terms of empirical parameters, the energy per particle of ANM is

$$e = E_0 + J_{sym}\delta^2 + L_{sym}\delta^2 x + \frac{1}{2}(K_0 + K_{sym}\delta^2)x^2 \qquad (4)$$

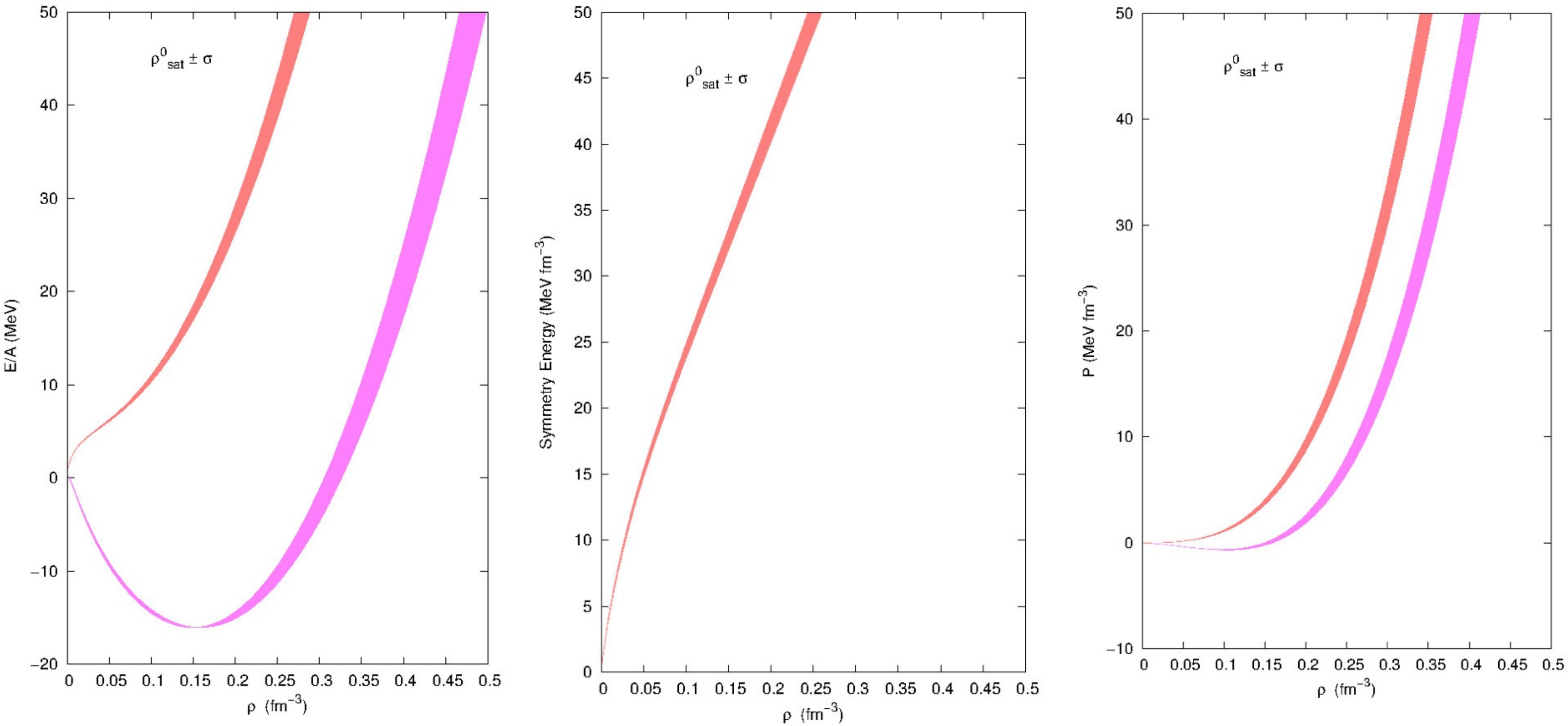
Homogeneous Nuclear Matter (HNM)

- One can determine the coefficients of this expansion in terms of empirical parameters $(\rho_0, E_0, K_0, J_{sym}, L_{sym}, K_{sym})$

$$
\begin{aligned}
t_0^{FG} &= \frac{3}{10m}\left(\frac{3\pi^2\rho_0}{2}\right)^{2/3} \\
a_{00} &= E_0 - t_0^{FG}(1+\bar{m}) \\
a_{10} &= -t_0^{FG}(2+5\bar{m}) \\
a_{20} &= K_0 - 2t_0^{FG}(5\bar{m}-1) \\
a_{02} &= J_{sym} - \frac{5}{9}t_0^{FG}\left[1+(\bar{m}+3\bar{\Delta})\right] \\
a_{12} &= L_{sym} - \frac{5}{9}t_0^{FG}\left[2+5(\bar{m}+3\bar{\Delta})\right] \\
a_{22} &= K_{sym} - \frac{10}{9}t_0^{FG}\left[-1+5(\bar{m}+3\bar{\Delta})\right]
\end{aligned}
$$

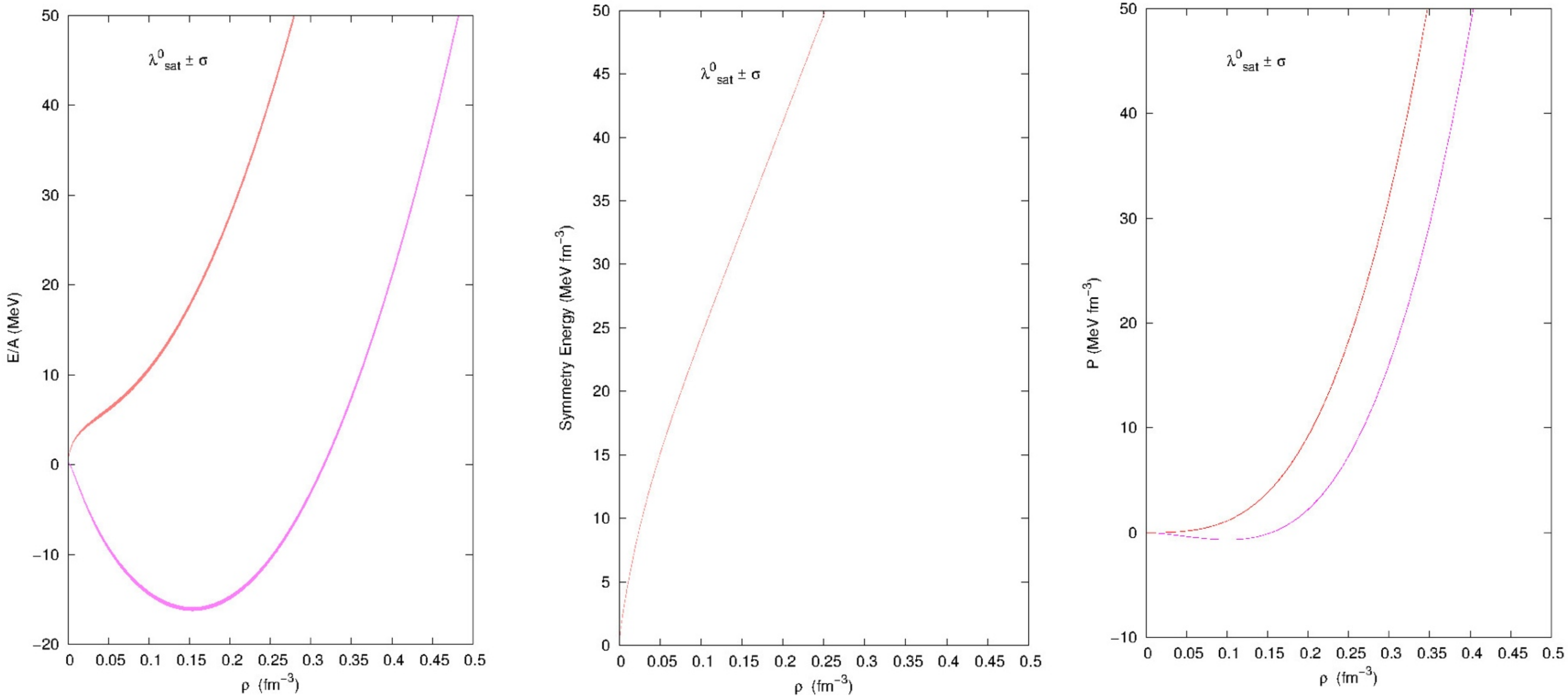
- $\bar{m}$ and $\bar{\Delta}$ are related to in-medium effective mass and isospin splitting of nucleon masses

RESULTS : HNM



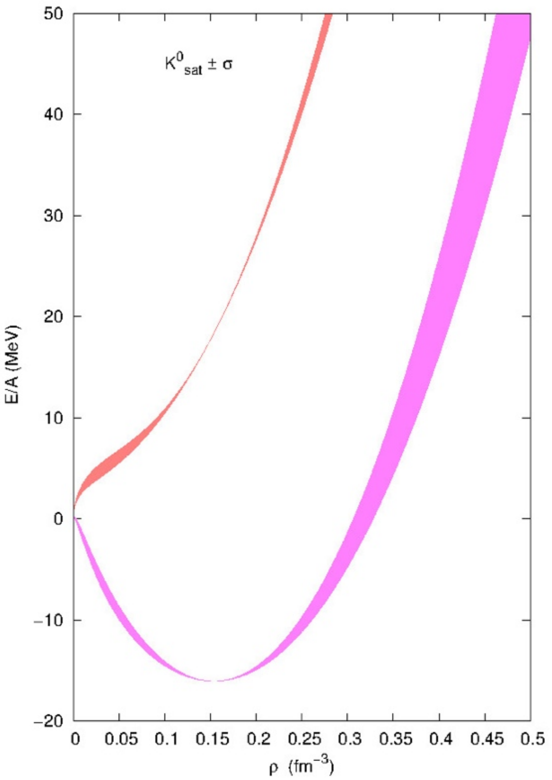
Effect of uncertainty in saturation density
on binding energy, symmetry energy and pressure

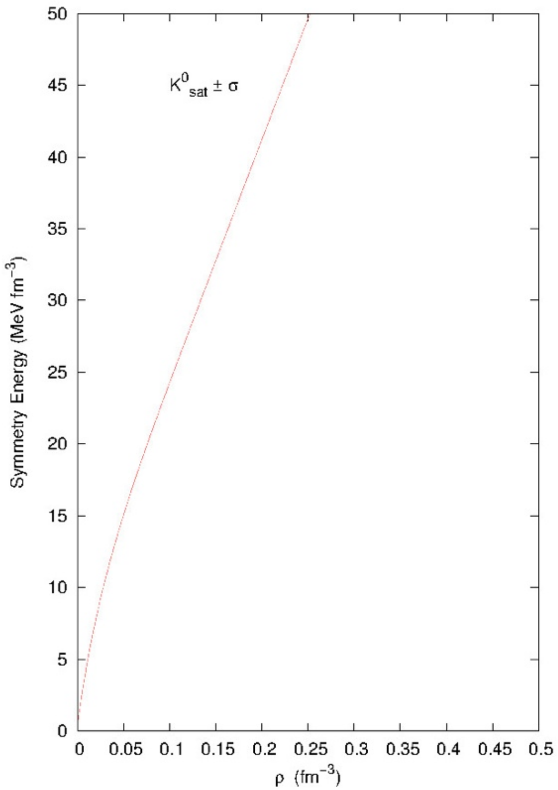
RESULTS : HNM

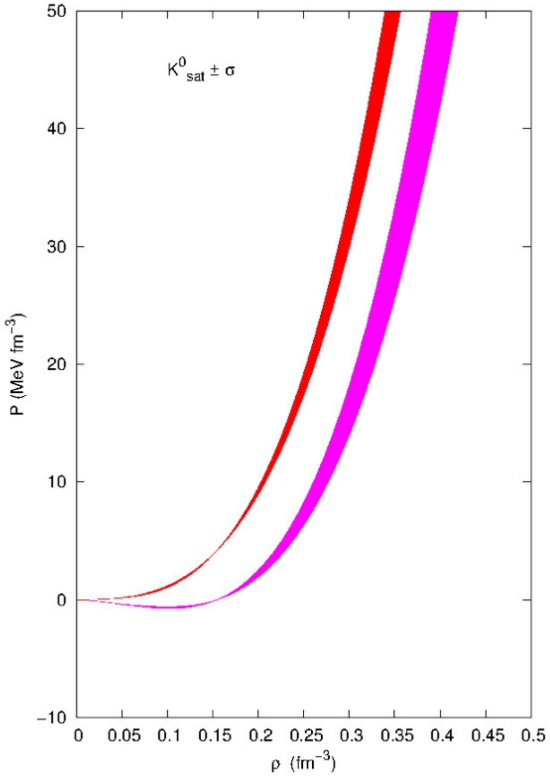


Effect of uncertainty in λ_{sat} on binding energy, symmetry energy and pressure

RESULTS : HNM

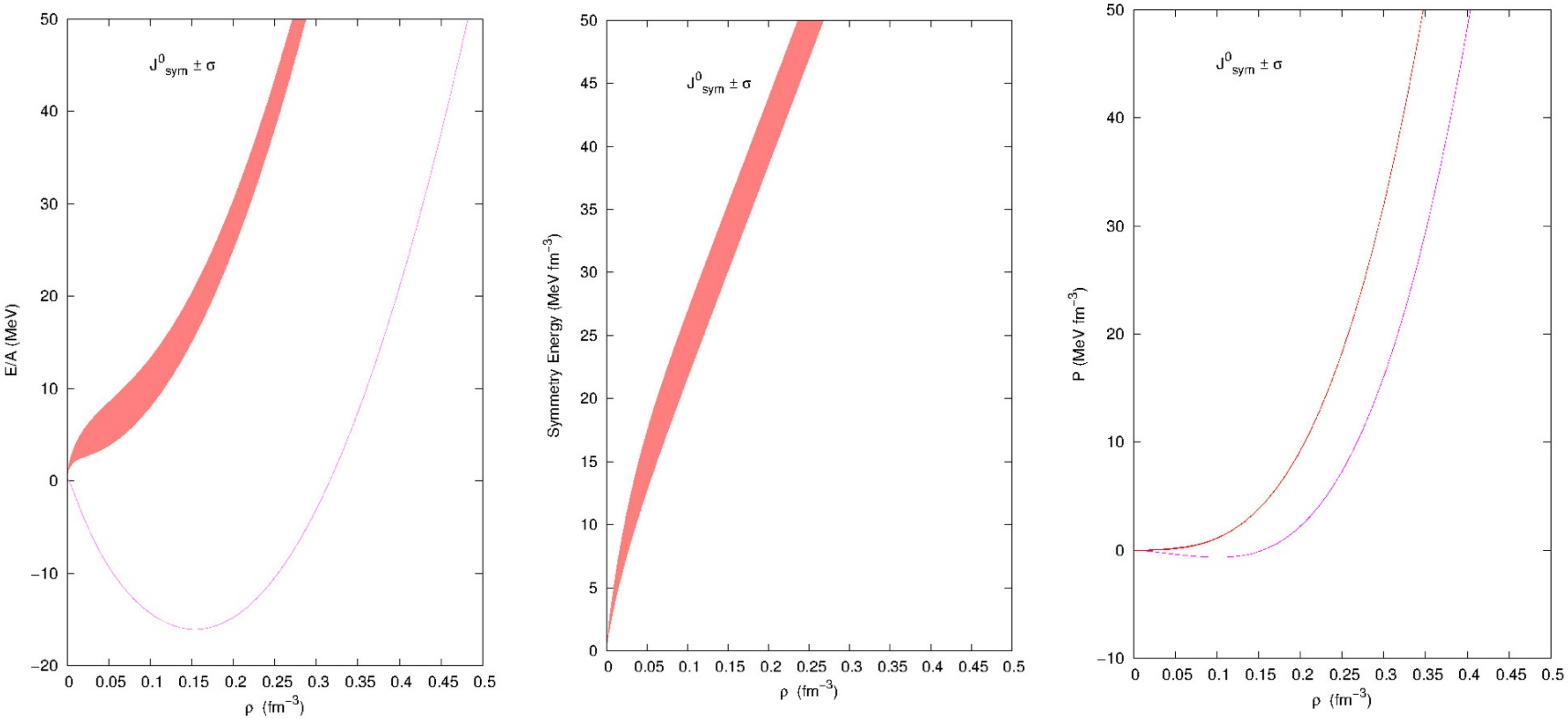






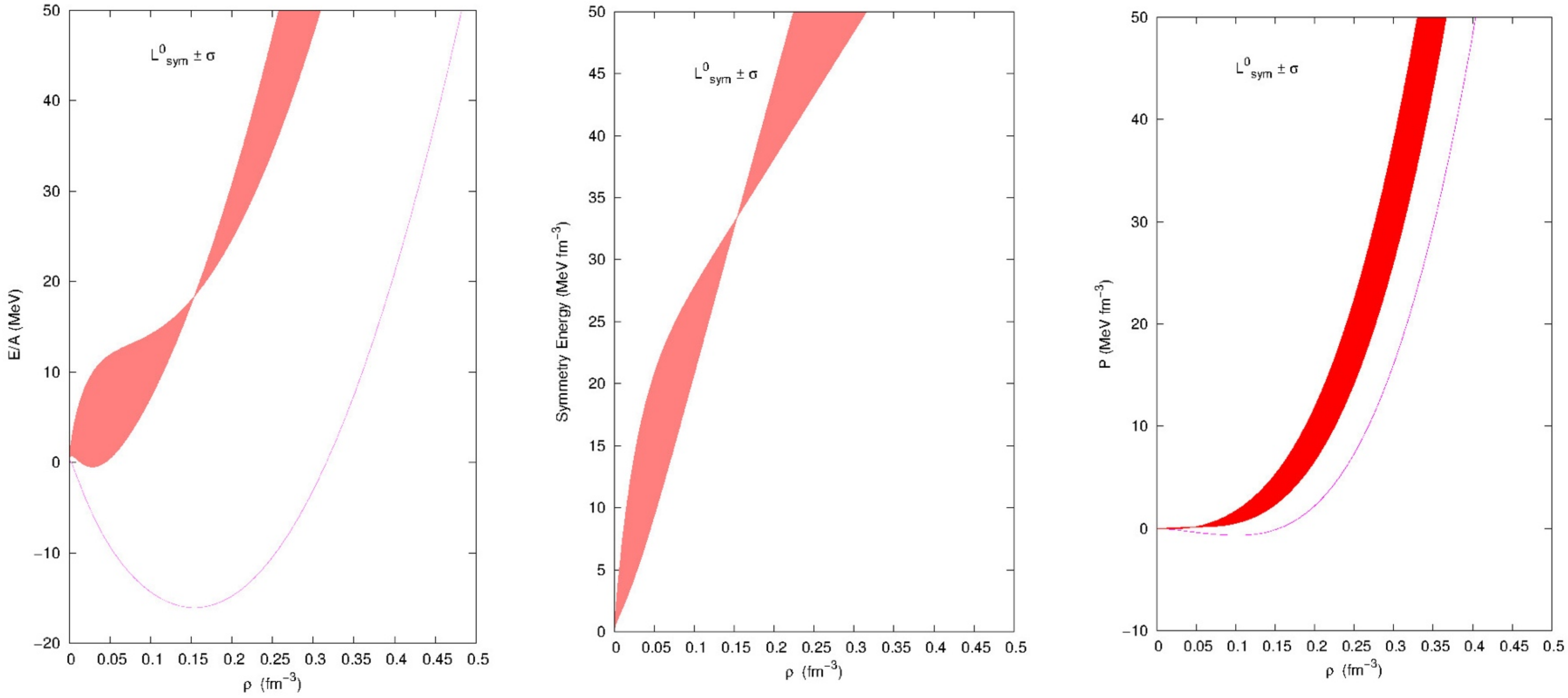
Effect of uncertainty in K_{sat} on binding energy, symmetry energy and pressure

RESULTS : HNM



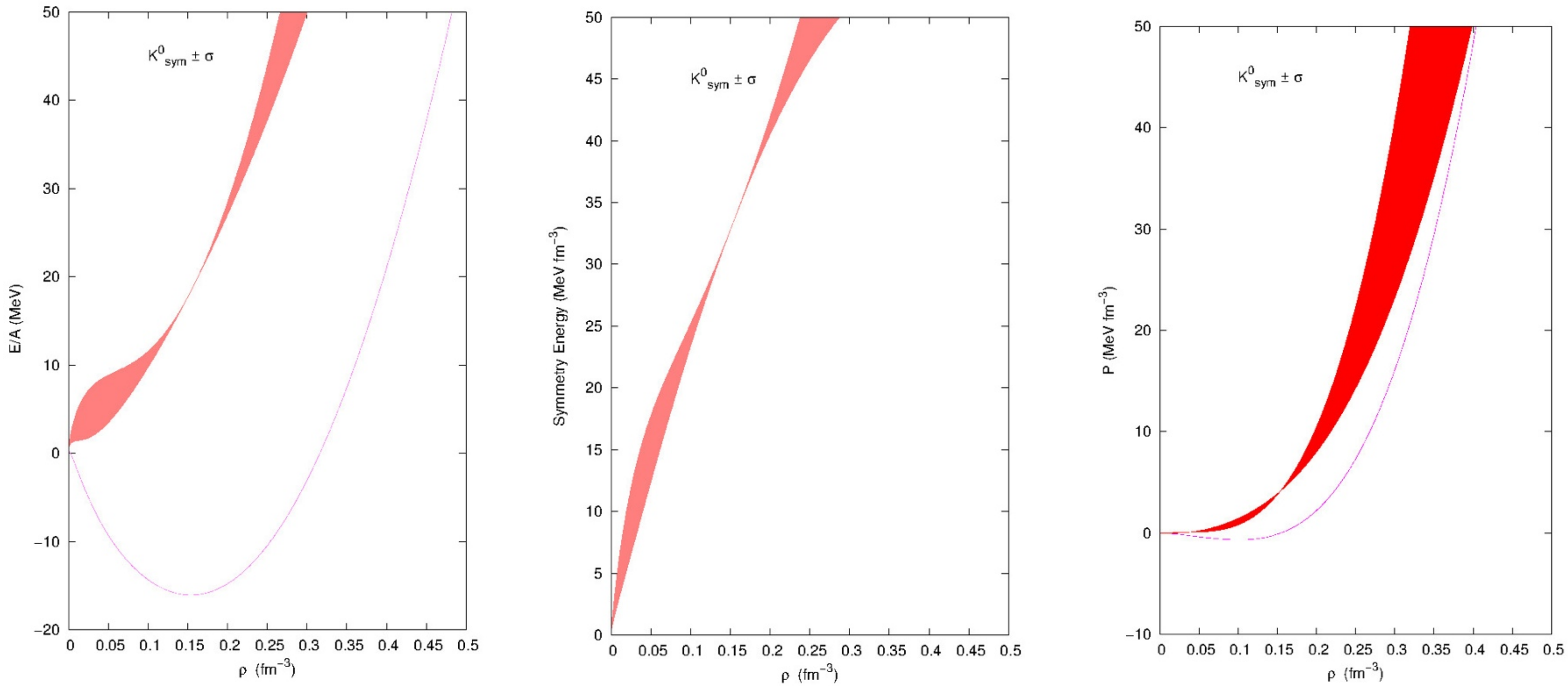
Effect of uncertainty in J_{sym} on binding energy, symmetry energy and pressure

RESULTS : HNM



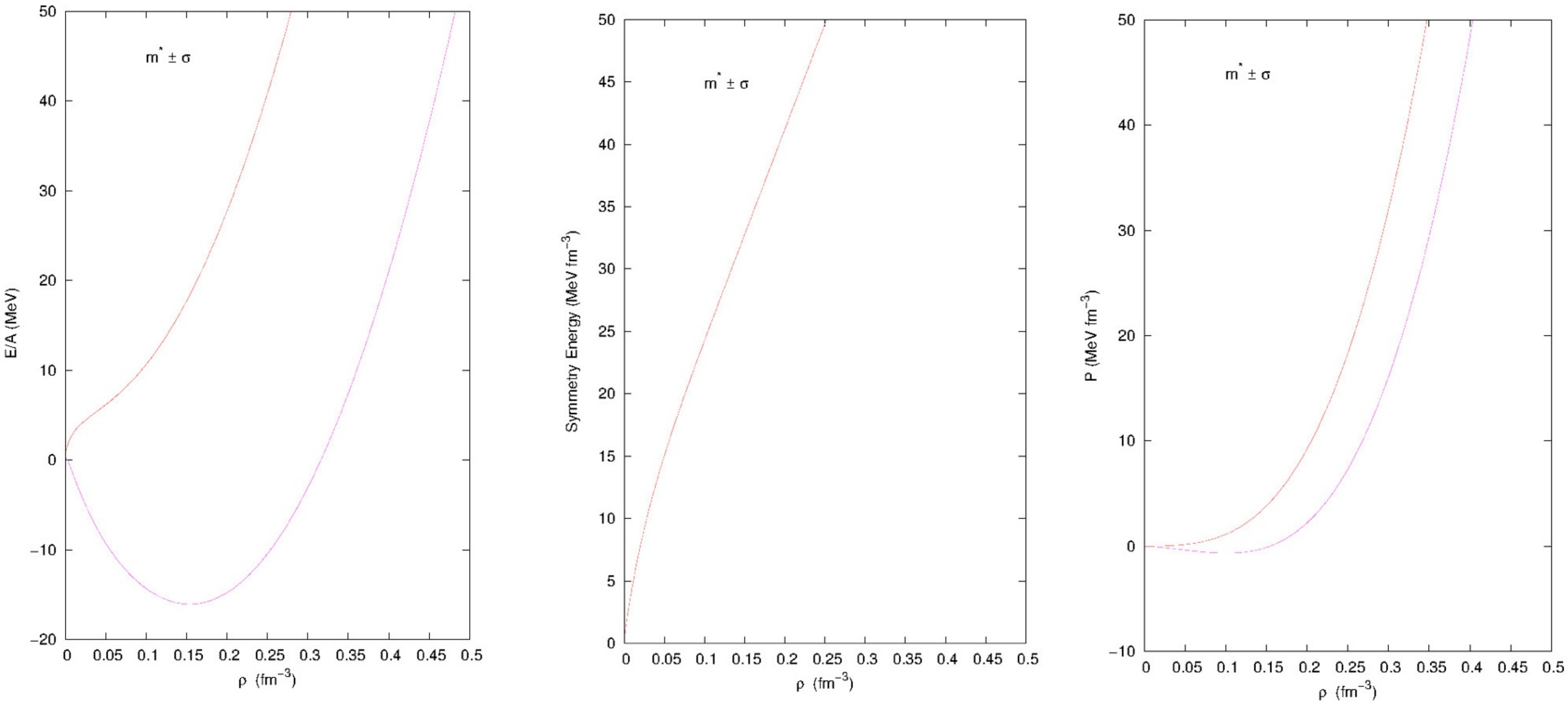
Effect of uncertainty in L_{sym} on binding energy, symmetry energy and pressure

RESULTS : HNM



Effect of uncertainty in K_{sym} on binding energy, symmetry energy and pressure

RESULTS : HNM



Effect of uncertainty in effective mass
on binding energy, symmetry energy and pressure

Inhomogeneous matter: Asymmetric nuclei

- Given a parametrized density profile $\rho_q(r)$, the energy of a nucleus

$$E = \int dr \mathcal{H}_{ETF}\left(\rho_q(r)\right)$$

- Fermi function is a reasonable choice for the density profile

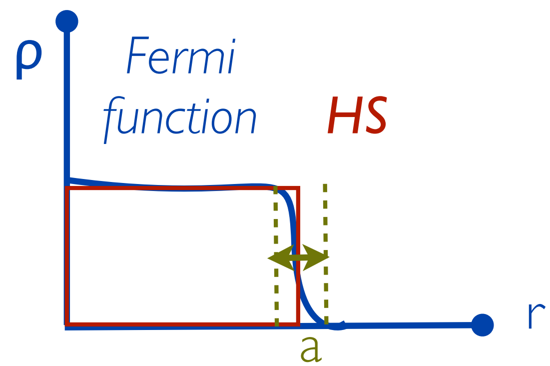
$$\rho_q(r) = \rho_0(\delta) F(r), F(r) = (1 + e^{(r-R)/a})^{-1} \tag{5}$$

where a is the diffuseness of the density profile

- saturation density of asymmetric matter

$$\rho_0(\delta) = \rho_0(\delta = 0)\left(1 - \frac{3L_{sym}}{K_{sat} + K_{sym}\delta^2}\right) \tag{6}$$

Papakonstantinou, Margueron, Gulminelli, Raduta, Phys Rev C 88 (2013) 045805



Inhomogeneous matter: Asymmetric nuclei

- Separating into the bulk and surface contributions,

$$E = E_b + E_s$$

- The bulk part $E_b = \lambda_{sat} A$, where the energy per particle

$$\begin{aligned} \lambda_{sat} &= \left. \frac{\mathcal{H}}{\rho} \right|_{\rho_0(\delta)} \\ &= e_{HNM}(x, \delta) \end{aligned}$$

where $x = \frac{\rho - \rho_0(\delta)}{3\rho_0(\delta)}$

Aymard, Gulminelli, Margueron
J. Phys. G 43 (2016) 045106

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Inhomogeneous matter: Asymmetric nuclei

- The surface energy can be separated into local and non-local terms

$$E_s = E_s^L + E_s^{NL}$$

$E_L = E_L$ [Empirical parameters],
$E_{NL} = E_{NL}$ [Empirical parameters $+ C_{fin} + C_{so}$]

- In the limit of purely local energy functional, the optimal configuration is a homogeneous hard sphere $a = 0$.
- The presence of NL terms in the functional results in finite diffuseness for atomic nuclei
- The diffuseness can be determined by $\frac{\partial E}{\partial a} = 0$

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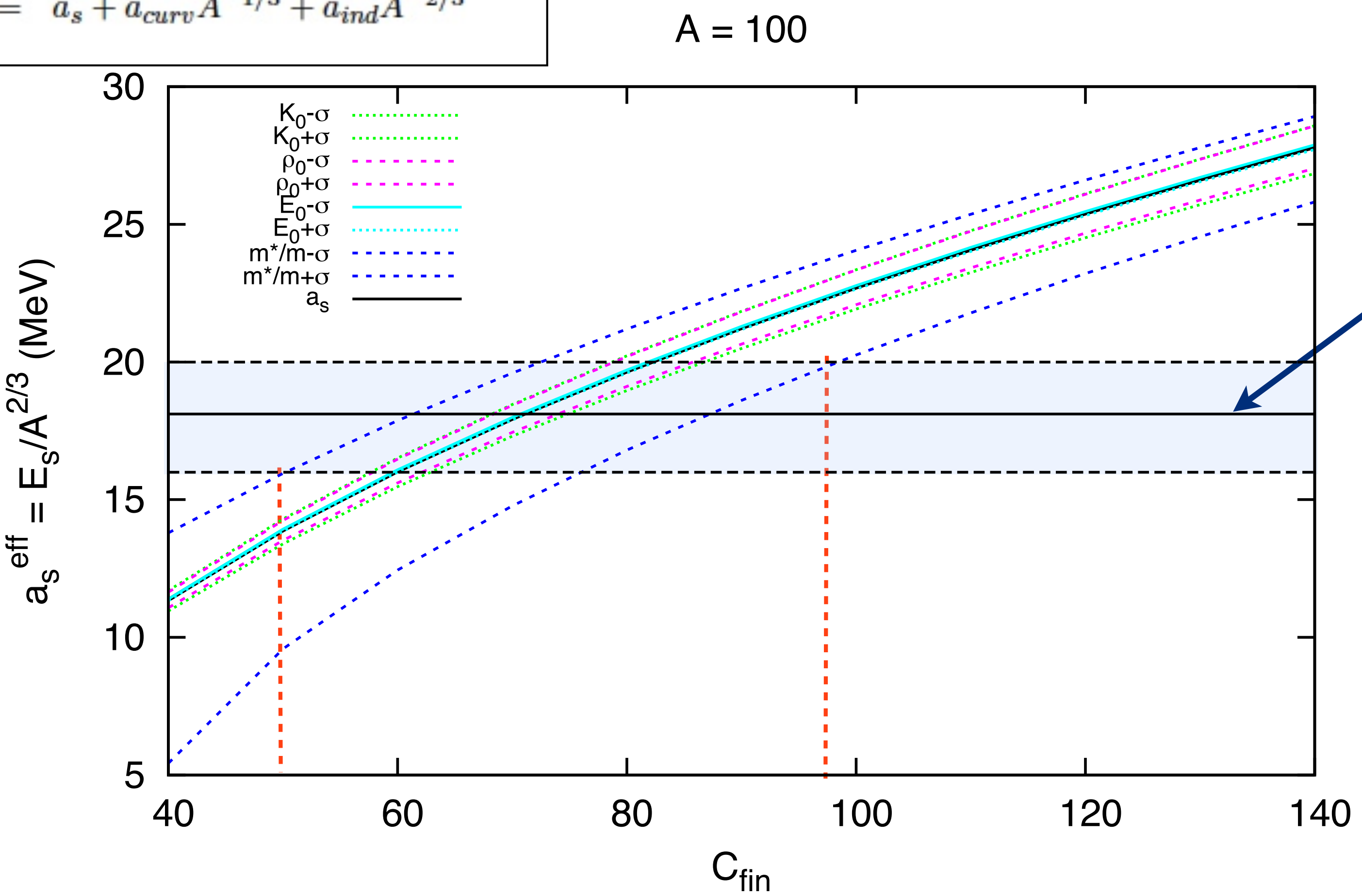
SURFACE ENERGY COEFFICIENT : ADDITIONAL CONSTRAINT

$$a_s^{eff} = \left(\frac{E}{A} - a_v\right) A^{-1/3}$$
$$= a_s + a_{curv} A^{-1/3} + a_{ind} A^{-2/3}$$

Danielewicz and Lee (2009)



$a_s^{eff} \approx 18.11 \pm 2$ MeV from nuclear mass data

Effect of uncertainty in ρ_{sat}, λ_{sat}, K_{sat}, m*/m on empirical value of C_{fin} using experimentally determined values of a_s^{eff} => $C_{fin} = 75 \pm 25$ MeV

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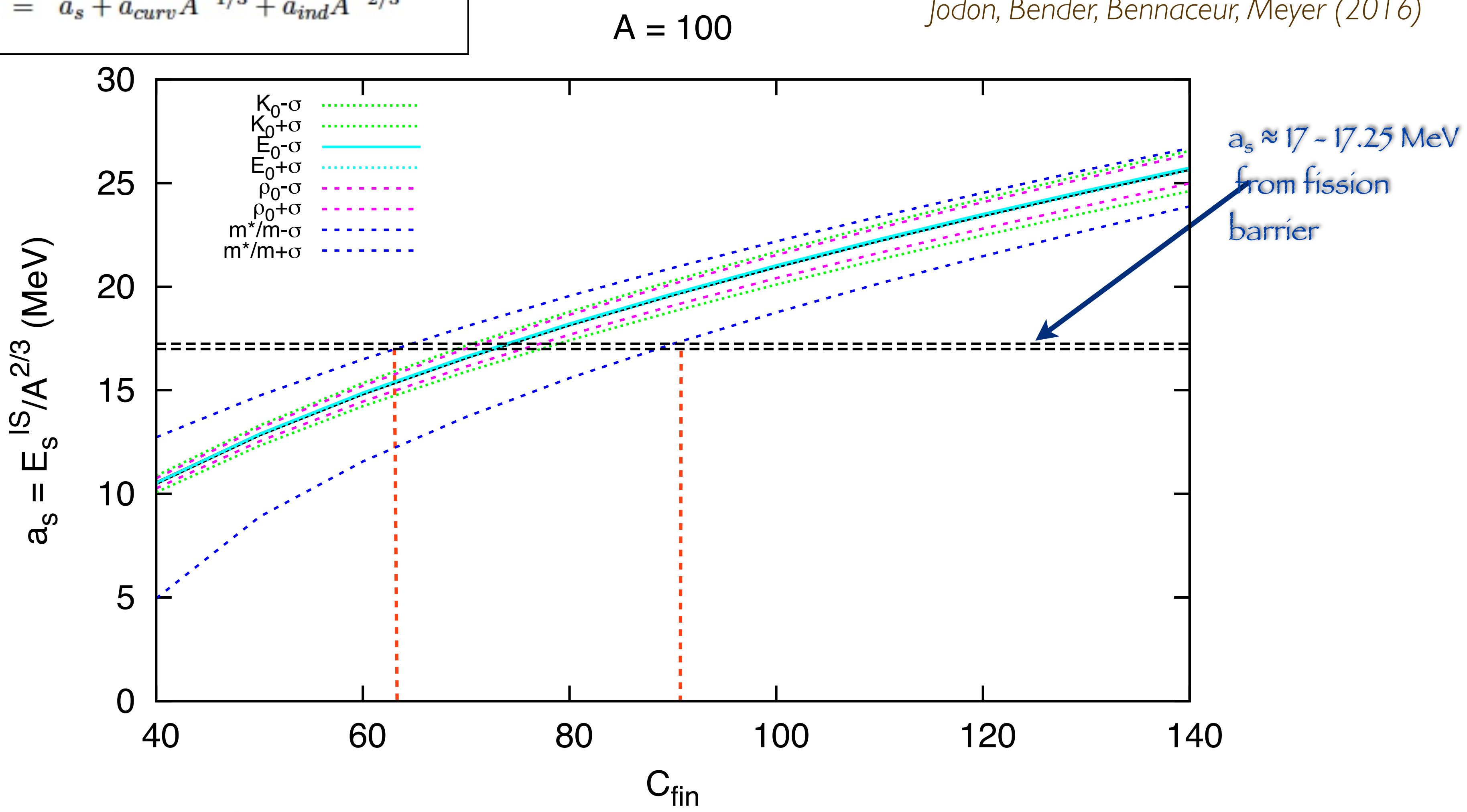
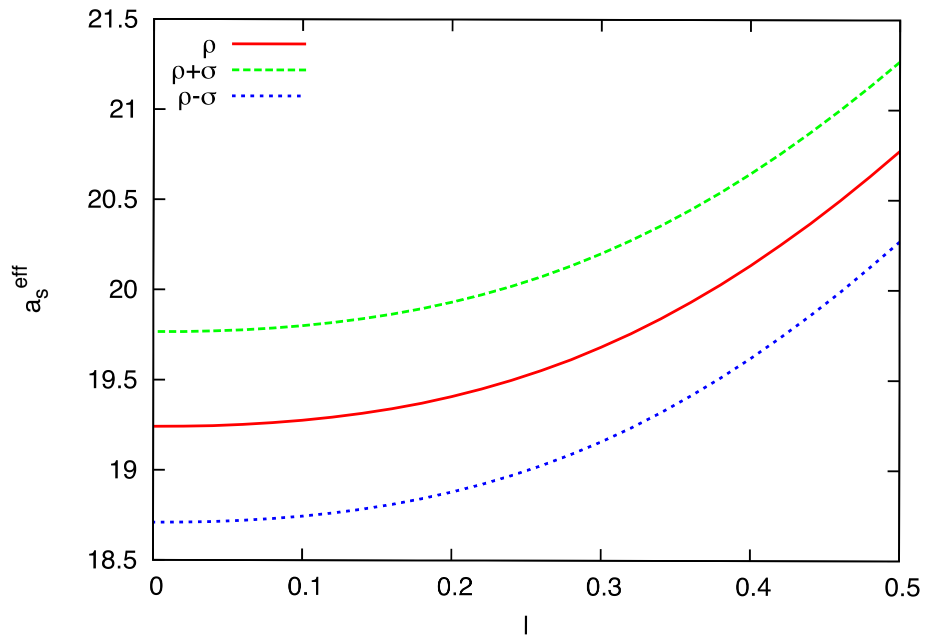
Jodon, Bender, Bennaceur, Meyer (2016)

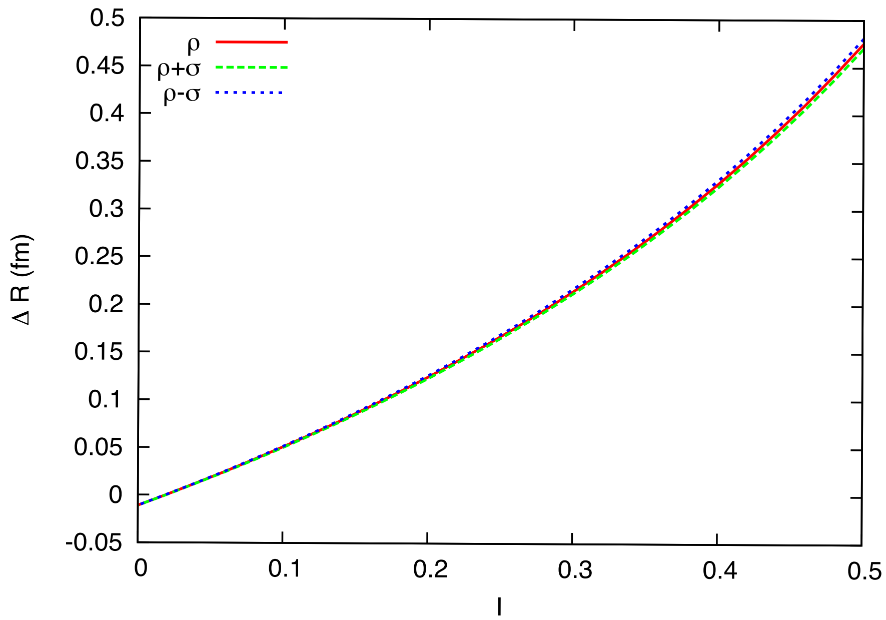


Effect of uncertainty in ρ_{sat}, λ_{sat}, K_{sat}, m^*/m on empirical value of C_{fin} using experimentally determined values of a_s => $C_{fin} = 77.5 \pm 12.5$ MeV

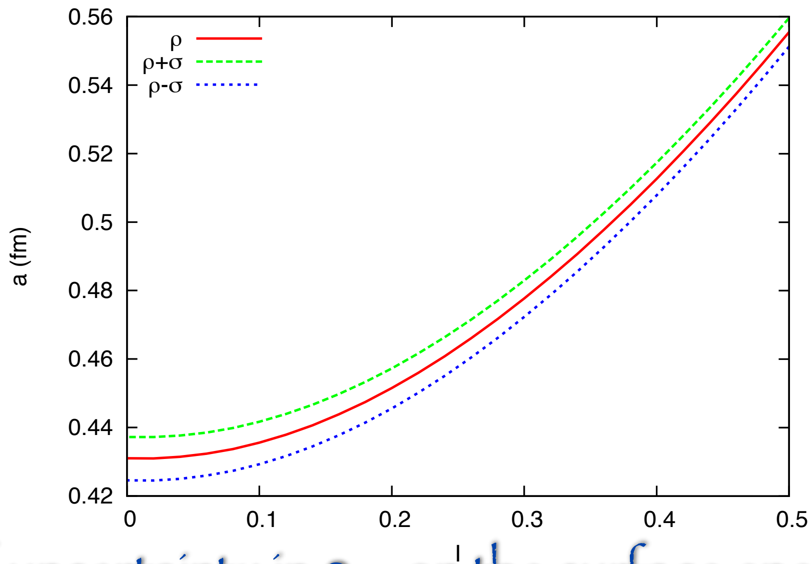
preliminary!

RESULTS : ASYMMETRIC NUCLEI





$$I = 1 - 2\frac{Z}{A}$$



$$\Delta R(a) = \Delta R_{HS}\left(1 + \frac{\pi^2}{3}\frac{a^2}{R_{HS}R_{HS,p}}\right)$$
$$\Delta R_{HS}(A,Z) = R_{HS}(A) - R_{HS,p}(Z)$$

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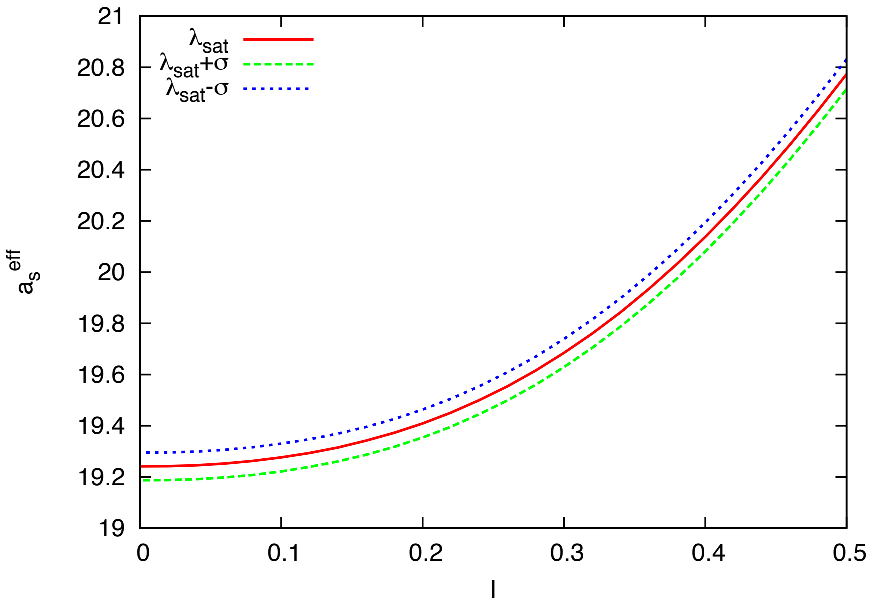
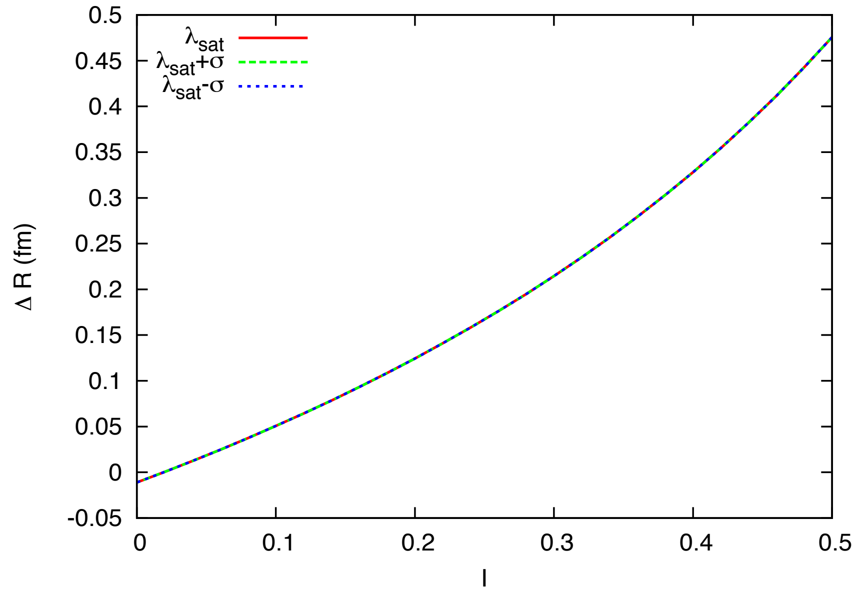
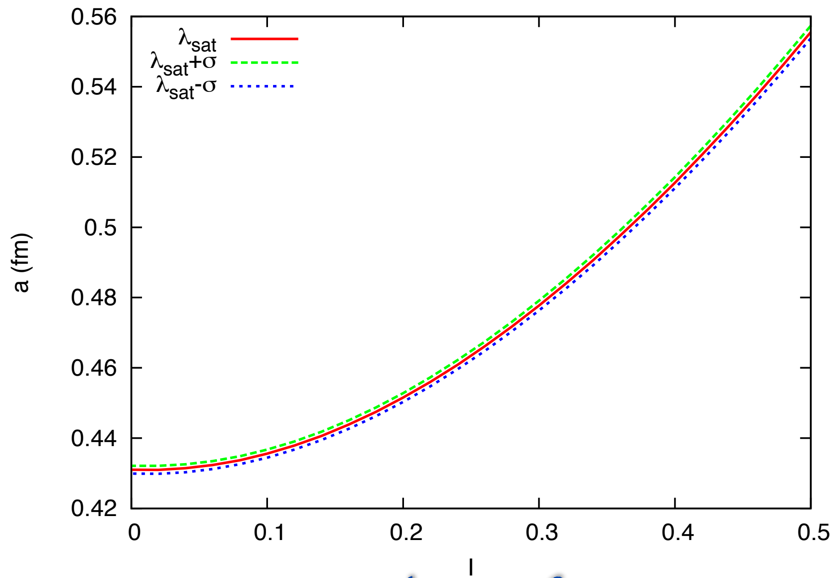
Effect of uncertainty in ρ_{sat} on the surface energy coefficient, neutron skin and diffuseness parameter

RESULTS : ASYMMETRIC NUCLEI

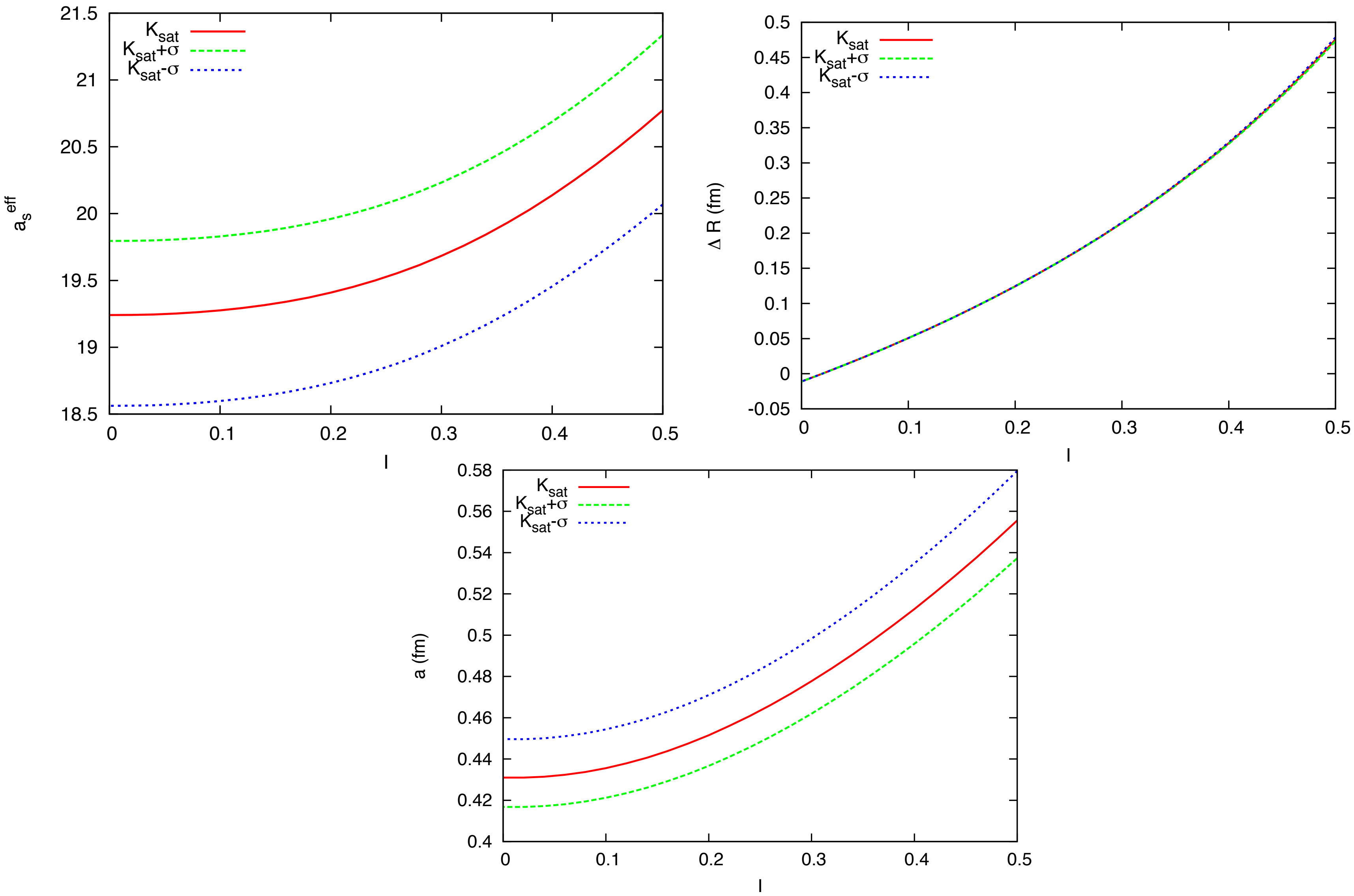






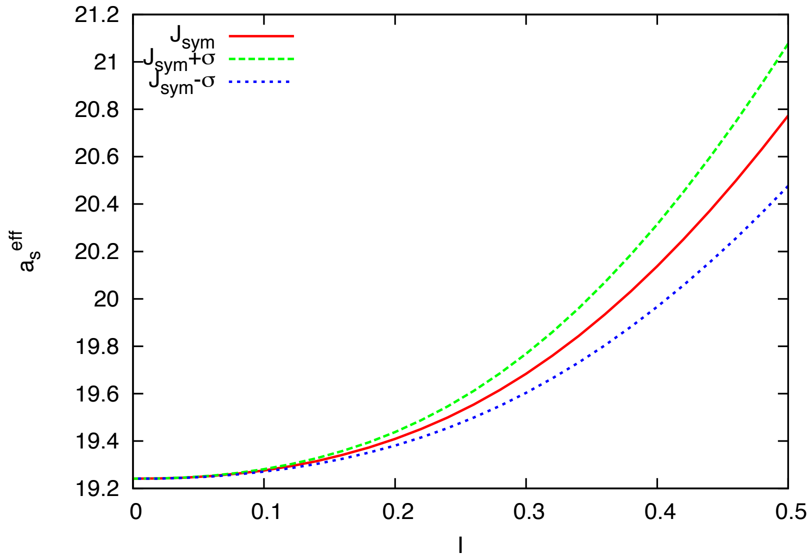
Effect of uncertainty in λ_{sat} on the surface energy coefficient, neutron skin and diffuseness parameter

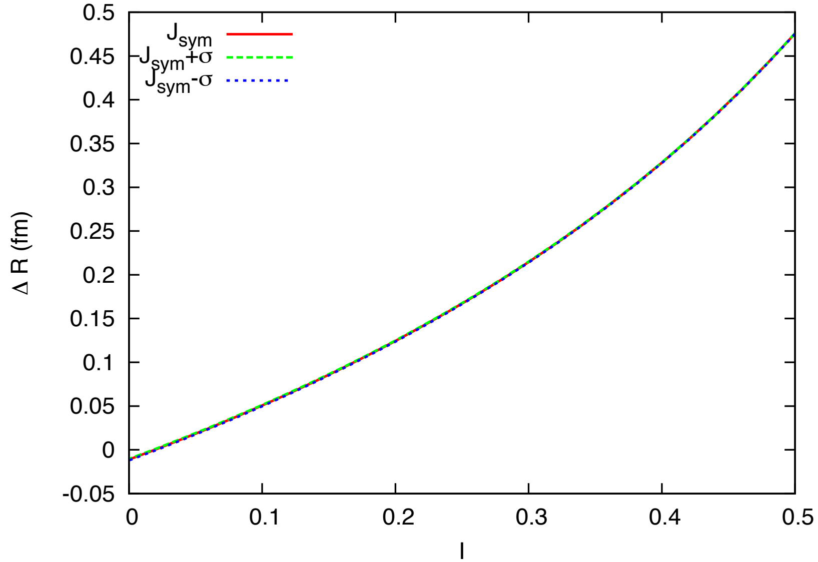
RESULTS : ASYMMETRIC NUCLEI

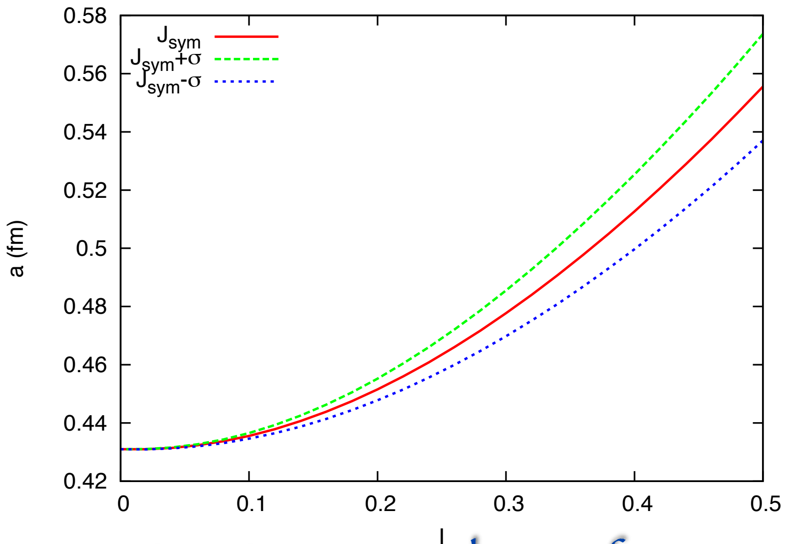


Effect of uncertainty in K_{sat} on the surface energy coefficient, neutron skin and diffuseness parameter

RESULTS : ASYMMETRIC NUCLEI



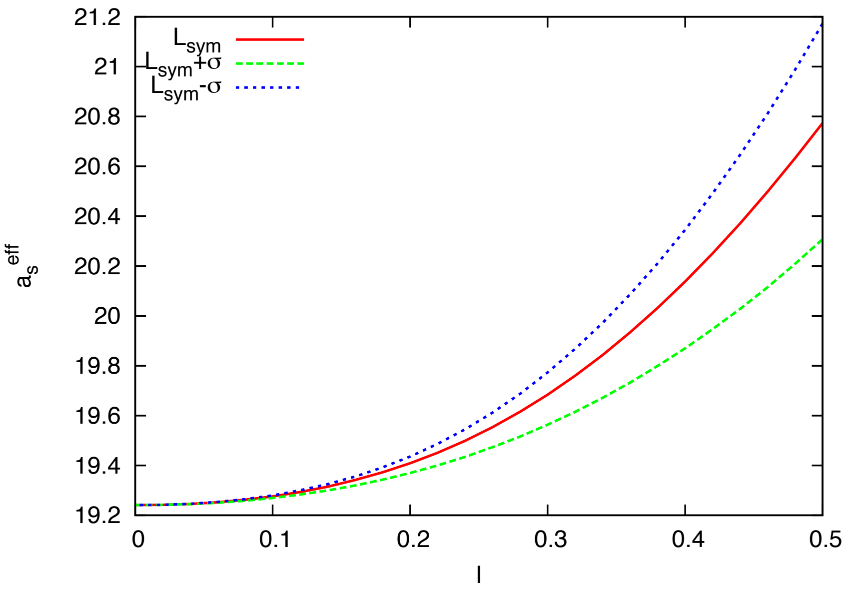


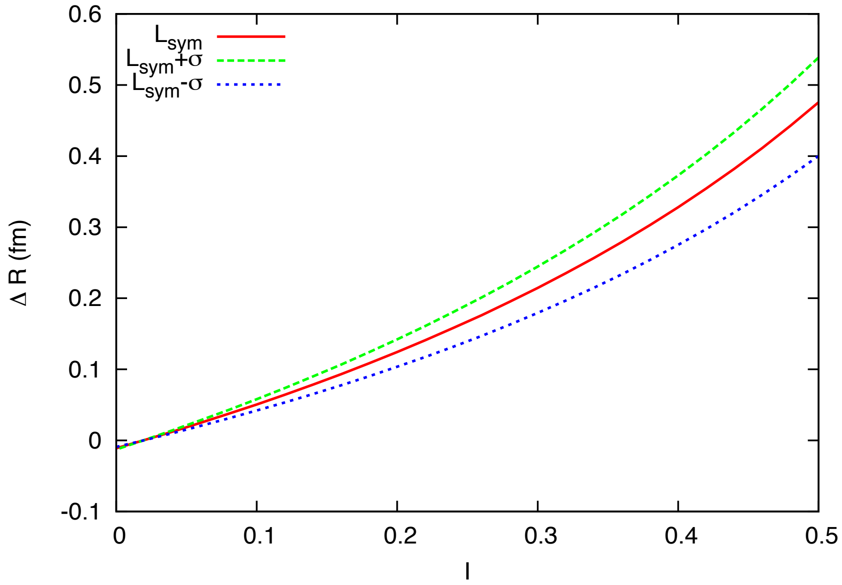


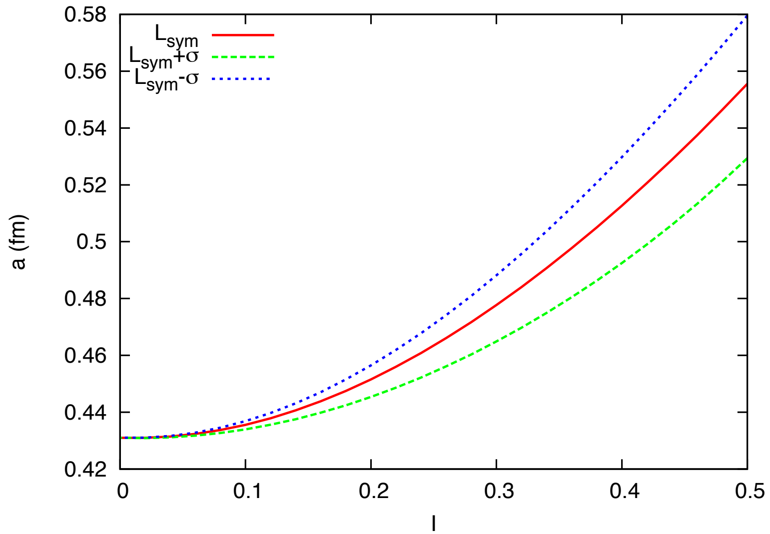
Effect of uncertainty in J_{sym} on the surface energy coefficient, neutron skin and diffuseness parameter

preliminary!

RESULTS : ASYMMETRIC NUCLEI



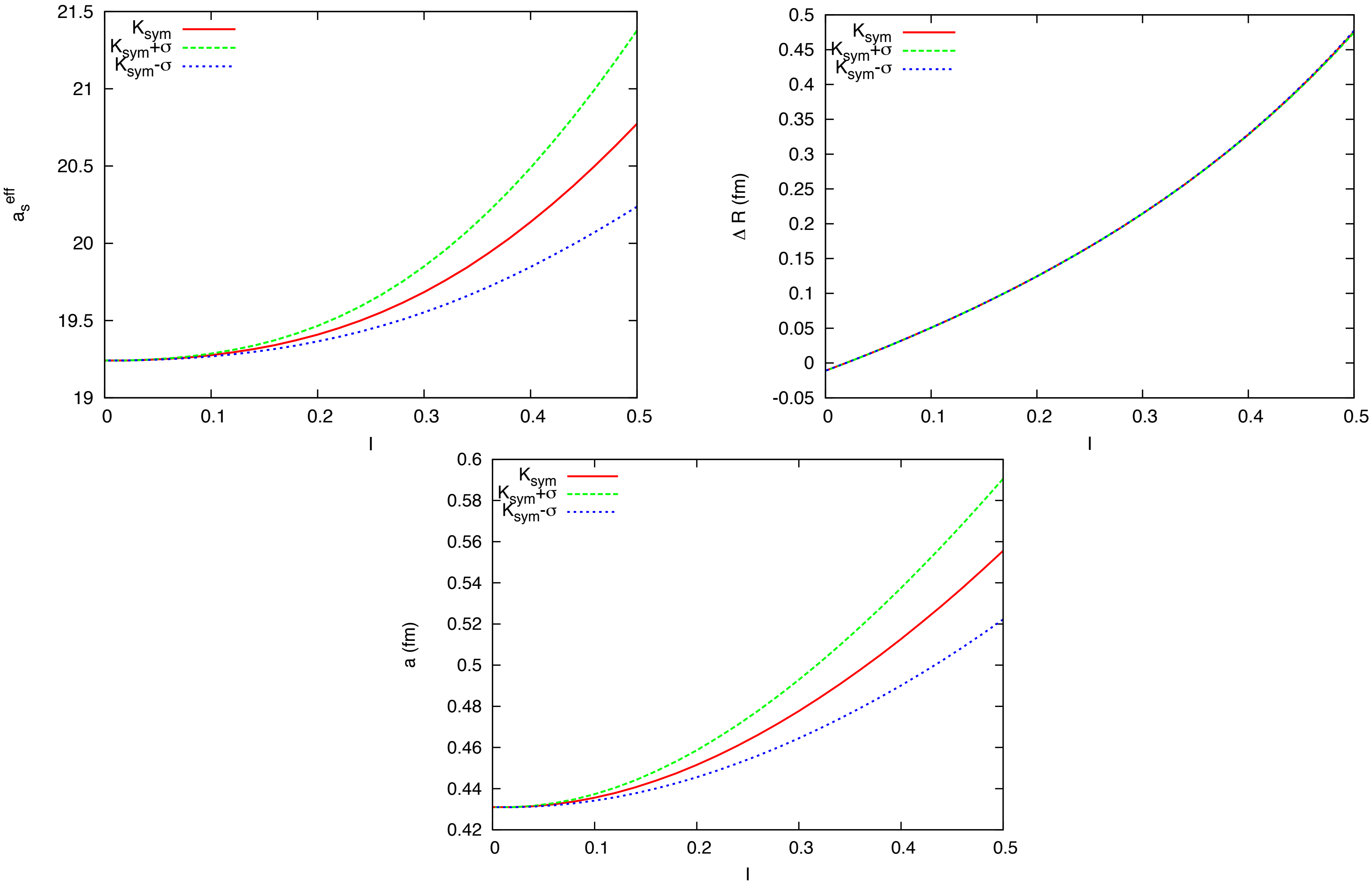




Effect of uncertainty in L_{sym} on the surface energy coefficient, neutron skin and diffuseness parameter

preliminary!

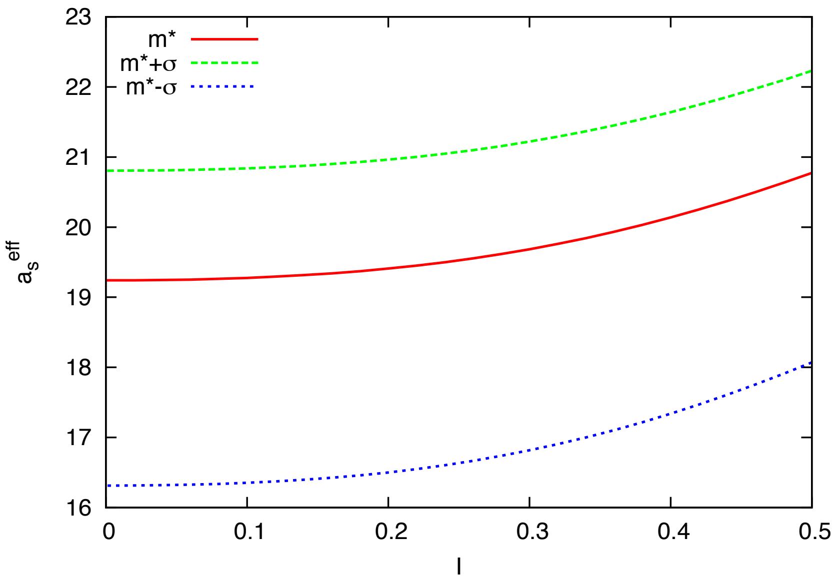
RESULTS : ASYMMETRIC NUCLEI

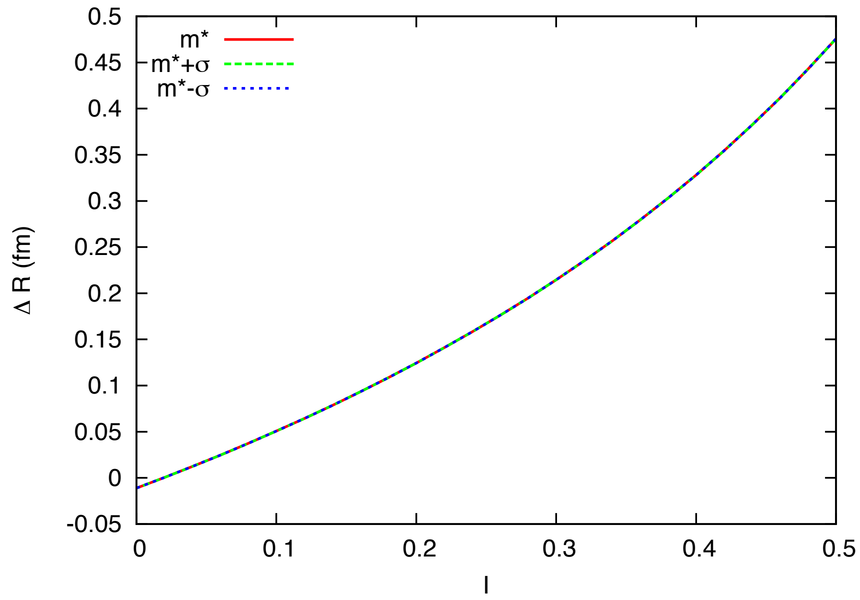


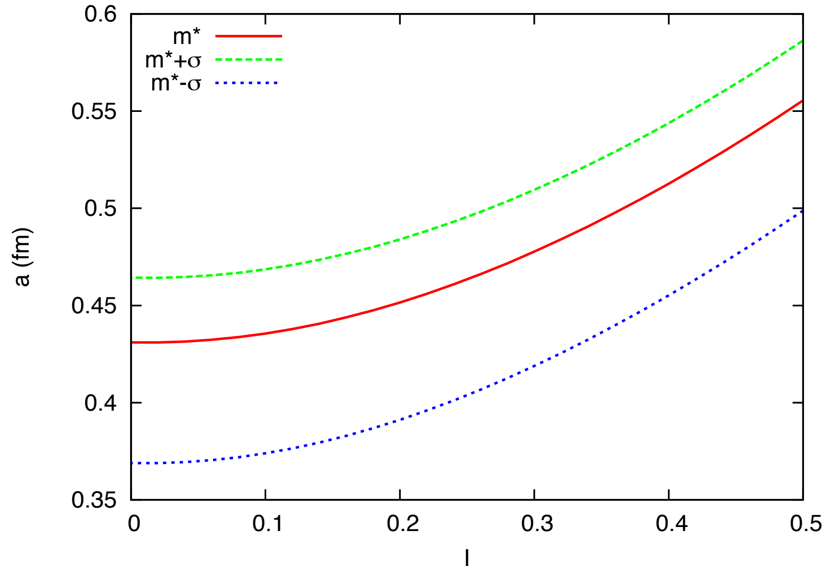
Effect of uncertainty in K_{sym} on the surface energy coefficient, neutron skin and diffuseness parameter

preliminary!

RESULTS : ASYMMETRIC NUCLEI



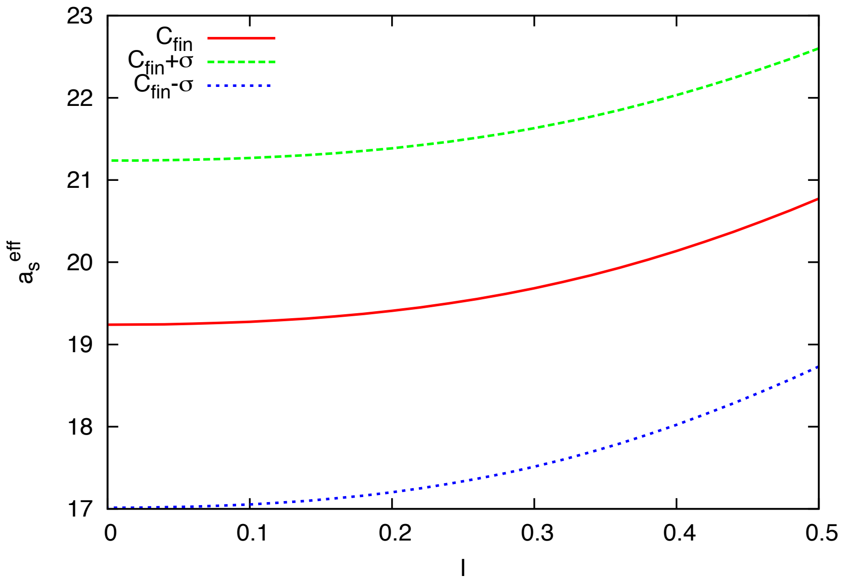


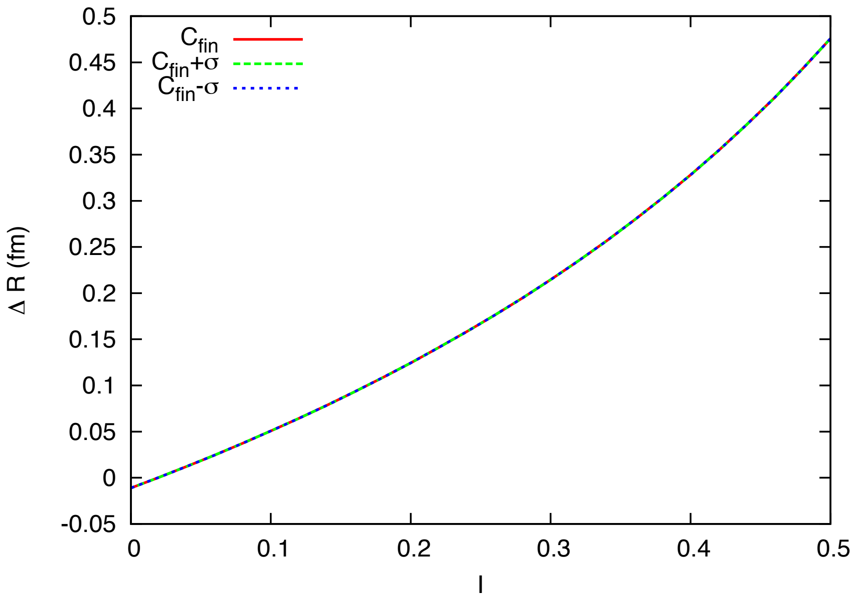


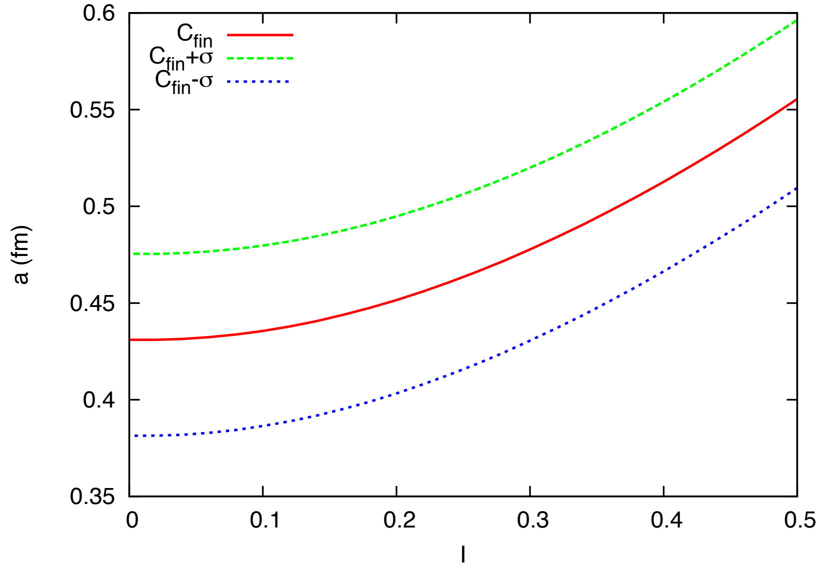
Effect of uncertainty in m* on the surface energy coefficient, neutron skin and diffuseness parameter

preliminary!

RESULTS : ASYMMETRIC NUCLEI



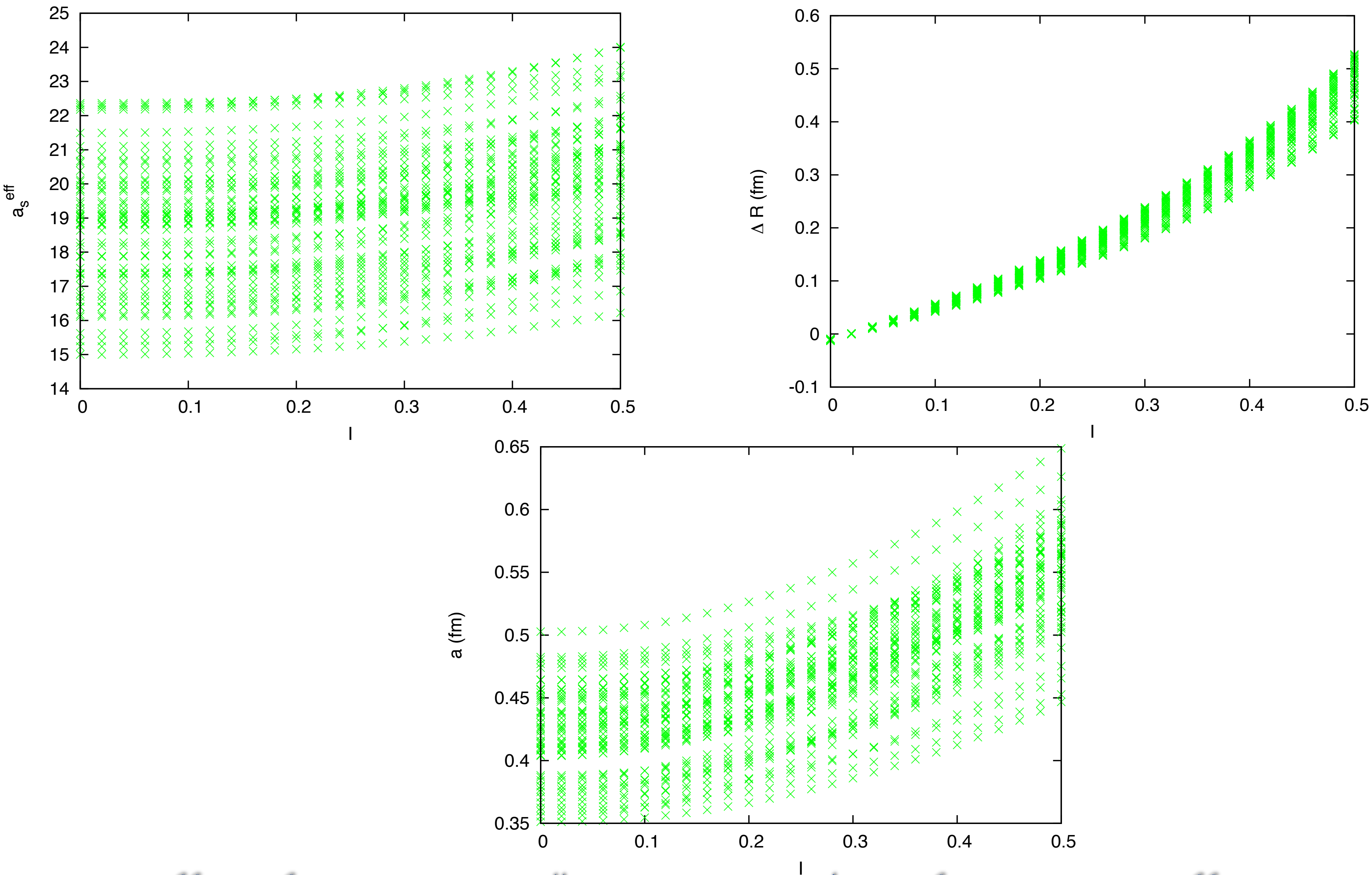




Effect of uncertainty in C_{fin} on the surface energy coefficient, neutron skin and diffuseness parameter

preliminary!

RESULTS : ASYMMETRIC NUCLEI



Effect of uncertainty in all parameters on the surface energy coefficient, neutron skin and diffuseness parameter

SUMMARY & PROSPECTS

- *Our aim is to develop a model independent (based on empirical constraints) "unified" EOS to describe core (homogeneous nuclear matter) + crust (asymmetric nuclei) to study properties of the crust-core interface*
- *We used DFT in the ETF approximation to construct an energy functional for HNM and clusterized matter*
- *In HNM, the coefficients of the energy functional are directly related to experimentally determined empirical parameters {ρ_{sat}, λ_{sat}, K_{sat}, J_{sym}, L_{sym}, K_{sym}} and m**
- *In clusterized matter, additional parameters appear, which are constrained using recent data for nuclear masses and fission barrier (C_{fin} depends on m*)*
- *The main uncertainty in a_s^{eff} comes from ρ_{sat}, K_{sat} and also m* (and C_{fin}). Although m* does not affect the properties of HNM, it affects surface energy of finite nuclei!*
- *Our results confirm that the neutron skin depends only on L_{sym}*
- *Work is in progress to apply these calculations to describe asymmetric nuclei in the outer crust and then to inner crust*
- *To study the influence of the uncertainty in empirically determined parameters on the crust-core transition*